

## Vinimetric: A Practical Rule for Calculating the General Term of an Arithmetic Progression

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**Abstract:** This paper presents an alternative for calculating the general term of an arithmetic progression. The so-called Vinimetric rule was discovered by Vinícius de Carvalho Oliveira in a 2nd year high school math class. This new approach is presented as a faster and simpler alternative to the conventional use of the formula. The work aims to validate and explore the educational potential of “Vinimetrica”, promoting discussions about new approaches to teaching mathematics and encouraging a creative and questioning school environment.

**Key words:** arithmetic progression, number sequence, mathematics education

### 1. Introduction

The discussion about how mathematical knowledge is produced is a topical issue: have concepts always existed and been discovered when humanity needed them, or did mathematicians invent numbers and formulas as tools for solving problems? According to the statement: “God created whole numbers. Everything else is the work of man” (Leopold Kronecker, 1823–1891), we can reflect on humanity’s need to continue searching for new ways to solve problems using mathematics as a tool.

New generations should be encouraged to study and understand how mathematical rigor provides discoveries that have a direct or indirect impact on various sectors of society. In view of this, mathematics education should serve as a starting point for training individuals committed to questioning concepts and theories, seeking different ways of thinking to develop reasoning and the ability to formulate scientific opinions and conclusions when faced with facts or experiments, thereby promoting the construction of knowledge.

“Students investigate when they seek, using their already constructed knowledge, to discover paths and decide which ones they should take to solve the problem, working collaboratively, relating ideas and discussing what should be done to reach the solution.” (ONUChic, 2008, p. 83).

Stimulating creativity and new approaches to solving problems contributes to the formation of citizens who think critically and are qualified to use mathematical tools to solve the challenges of their daily lives.

Returning to the initial question, about the construction of mathematical knowledge, regardless of the point of view, whether you think like Penrose who says: “mathematics has always existed and we are exploring the discoveries” or like Wolfram, who succinctly argues: “mathematics is just a tool invented by man to understand

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phenomena”, the classroom should be an environment conducive to discoveries or inventions, because the study of this science acquires greater meaning for students when the methodologies employed favor investigation and the search for new ways to solve a problem.

## 2. Arithmetic Progression and the Emergence of Vinimetrics

The history of mathematics is marked by stories about how discoveries were driven by classroom experiences. One curious fact is the famous story of how Gauss, while still a child, performed the sum of the integers from 1 to 100 in an unusual way.

Johann Carl Friedrich Gauss was a German mathematician who lived in the Kingdom of Hanover, Brunswick. He was born on April 30, 1777, and died on February 23, 1855. In his 77 years of life, he became known as the prince of mathematics due to his influential contributions to various areas of this science.

According to a famous story, the principal of the school where Gauss studied asked his ten-year-old pupils to add up the integers from 1 to 100. Within a few minutes, Gauss had determined the correct result. The reasoning he used is now the formula for calculating the sum of the terms of an arithmetic progression.

“Arithmetic progression is any sequence of numbers in which the difference between each term (starting from the second) and the previous one is constant. This constant difference is called the ratio of the progression.” (Dante, 2017, p. 213). We use the acronym A.P. to refer to arithmetic progression.

Currently, the formula used to calculate the  $n$ th term of an A.P. is given by:

$$a_n = a_1 + (n - 1)r \quad (1)$$

where:

$a_1$  is the first term;

$r$  is the ratio;

$n$  is the position of the term in the sequence.

Curiously, in April 2024, during a mathematics class in the second year of high school at Escola Estadual Embaixador José Bonifácio, located in Barbacena, MG, student Vinicius de Carvalho Oliveira explored a novel approach to calculating any term of an arithmetic progression, diverging from the conventional application of the formula.

This study analyzes the rule he proposed, called the “Vinimetric Rule”, seeking to validate its use as an alternative method for obtaining the  $n$ th term of an arithmetic progression, representing an innovative variation of the classic formula.

After conducting the study and validating the rule, we propose its adoption as a didactic alternative for teaching mathematics in basic education, emphasizing the importance of fostering a constructive, creative, and inquisitive school environment.

According to the National Common Core Curriculum (BNCC), the knowledge object arithmetic progression is part of the mathematics curriculum in basic education and can be addressed in any grade of secondary education, depending on the curriculum plans of each school network or institution. During the presentation of this content to the second-year class, several examples of the application of the formula for the general term of an arithmetic progression were discussed. To better understand the concept, let us analyze the pattern illustrated in the Figure 1:

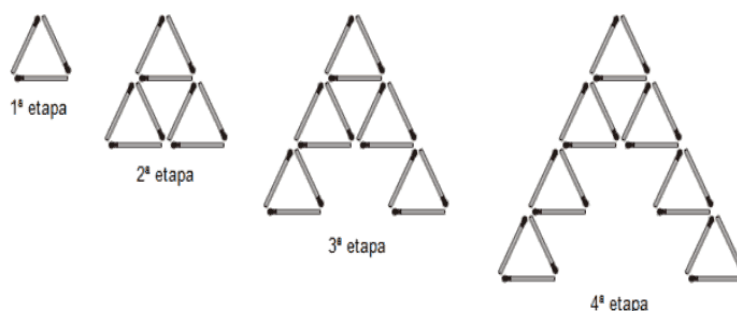


Figure 1 Pattern with matchsticks<sup>1</sup>.

The sequence below gives us the number of sticks used in each step:

$$3, 9, 15, 21, \dots \quad (2)$$

Add six to each term, starting at 3, to find the next term in the sequence.

Let's take a look at the number sequences below:

$$3, 5, 7, 9, \dots \quad (3)$$

Add two to each term, starting at 3, to find the next term in the sequence.

$$8, 11, 14, 17, \dots \quad (4)$$

Add three to each term, starting at 8, to find the next term of this number sequence.

All the numerical sequences exemplified above are called arithmetic progressions, because the difference between each term (starting from the second) and the previous one is constant.

Using the example of the matchsticks in Figure 1, let's determine the number of matchsticks needed to build the 23rd stage by calculating  $a_{23}$ .

We have an arithmetic progression (A.P.) where  $a_1 = 3$  is the number of toothpicks used in the first stage,  $a_2 = 9$  is the number of toothpicks used in the second stage, and so on,  $a_{23}$  will be the number of toothpicks used in the twenty-third stage.

$$3, 9, 15, \dots \quad (5)$$

$$a_n = a_1 + (n - 1)r$$

$$a_1 = 3;$$

$$n = 23;$$

$$r = 6$$

Substitute the values into the formula for the general term:

$$a_{23} = 3 + (23 - 1)6$$

$$a_{23} = 3 + (22)6$$

$$a_{23} = 3 + 132$$

$$a_{23} = 135$$

Using the rule of thumb, the above calculation would be greatly simplified.

The method suggested by student Vinícius is innovative and has never been documented before; at least, no records of it exist in the current bibliography. This study was initiated to prove its validity, as verifying a few

<sup>1</sup> Available online at: <https://guiadovestibulinho.com.br/wp-content/uploads/2018/12/6.png?x13558>.

examples does not ensure generalization to all terms of all arithmetic progressions.

After demonstrating the validity of this discovery through several examples, a group was formed comprising students Vinícius, Vitor, and Nicolas, who were eager to validate the method as a new practical rule, called Vinimetric, under the guidance of master teacher Thaís Presotti de Almeida Machado.

Just as the young Gauss offered a fresh perspective on summing the numbers from 1 to 100, student Vinícius proposed a new, much simpler method for performing such calculations, as exemplified below:

Example 1: Consider the arithmetic progression.

To calculate the 23rd term of the sequence, the student proceeded as follows:

*We have  $n = 23$*

*As 3 is the unit digit  $n$ , we consider the third term of the sequence ( $a_3 = 8$ ) to be the unit digit of  $a_{23}$ . The tens digit of  $n$  multiplied by the ratio results in the tens digit of  $a_{23}$ .*

*So, since:  $2 \times 3 = 6$  we have that  $a_{23} = 68$ .*

Example 2) Consider the PA 3, 7, 11, 15, ...

The student calculated the 39th term of the sequence as follows:

*We have  $n = 39$*

*As 9 is the units digit of  $n$ , we consider it the ninth term of the sequence.*

*To do this, we need to continue writing the next terms:*

*3, 7, 11, 15, 19, 23, 27, 31, 35, 39.*

*$a_9 = 35$*

*The unit digit of  $a_9$  (5) will be the unit digit of  $a_{39}$ . The tens digit of  $a_9$  (3) is reserved.*

*The tens digit of  $n = 39$  multiplied by the ratio,  $r = 4$ , added to the tens digit of  $a_9$ , 3.*

*$3 \cdot 4 = 12$*

*$12 + 3 = 15$*

*5 is the tens digit of  $a_{39}$*

*1 is the tens digit of  $a_{39}$*

By manipulating the formula currently in use, we concluded that the Vinimetric rule is a variation of its application, making it a valid and more efficient method for calculating the general term of an arithmetic progression.

The validity of the proposed rule was demonstrated by adapting the well-known formula for determining the general term of an arithmetic progression, thereby confirming its accuracy and practical applicability.

*Let  $n = 10.k + q$  be the  $n$ th term of an arithmetic progression, with  $n, k, q \in \mathbb{N}$  and  $1 \leq q \leq 10$*

*We have that  $a_n = a_1 + (n-1)r$ , from which it follows that*

$$a_n = a_1 + (10k + q - 1)r$$

$$a_n = a_1 + 10kr + (q-1)r$$

$$a_n = a_1 + (q-1)r + 10Kr$$

*How:*

The rule of thumb can therefore be used to calculate the  $n$ th term of any term in an arithmetic progression.

## 5. Conclusion

The Vinimetric rule represents an innovative alternative for calculating the general term of an arithmetic

progression. Its practicality ensures agility and offers a promising tool to assist students and teachers in solving problems in school activities and assessments.

Through this work, we introduce this novel method to the mathematical community, fostering discussions about the importance of cultivating a constructive, creative, and inquisitive school environment.

Moreover, we envision that, by encouraging critical thinking, students can develop an interest in the mathematical rigor of proofs, thereby promoting the continuity of mathematical knowledge and discoveries. These contributions have the potential to generate a significant impact on various sectors of Society.

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