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Effects of Spatial Price Discrimination on Output, Welfare, and Location: Comments and Extension

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Abstract: According to Hwang-Mai (1990), a monopolist facing a fixed number of markets with linear demands can yield smaller output and yet greater welfare under discriminatory rather than nondiscriminatory mill pricing policy. Unfortunately, this is an untenable proposition flawed by an invalid simulation. We find that within the confines of their simulation parameters it is the nondiscriminatory pricing policy that yields strictly greater welfare than does discriminatory pricing with no regulation. This does not necessarily mean that the Hwang-Mai conclusion is untenable, but it does reveal the risk of careless simulation that leads to nowhere.

Key words: uniform mill pricing, discriminatory pricing, endogenous location
JEL codes: R32, R11, D40

1. Introduction

Extending along the lines of some pioneer works, e.g., by Dupuit (1861), Pigou (1929), and Robinson (1933), on output and welfare effects of price discrimination, Greenhut and Ohta (1972) find that a spatial monopolist practicing discriminatory pricing produces greater output than he would under simple monopoly pricing. Willam Holahan (1975) added the analysis of welfare under alternative conditions of discriminatory and nondiscriminatory mill pricing. Martin Beckman (1976) also added another issue of spatial monopolistic competition, which limits an individual firm's market area by spatial entries, thereby negating Greenhut-Ohta's strictly positive output effect of discrimination.

Hong Hwang and Chao-Cheng Mai (1990) explores the Robinson output theorem and the Schmalensee welfare theorem under these alternative spatial pricing policies. They tried to show that their theorems on welfare would be unscathed in a spatial economy under some particular parameters. These parameters, however, do not support these theorems. Furthermore, only simple monopoly pricing policy proves to be superior under their parameters.

2. Hwang and Mai Model

Consider two separated markets on a line. The distance between the markets is denoted by s. A firm sells both markets by charging one of the following: an f.o.b. mill price, a uniform price and discriminatory prices. With an f.o.b. price policy the firm sets a mill price m, and transport r costs are paid by the consumers. Under a

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uniform-delivered-pricing policy, the firm charges all consumers the same delivered price regardless of the delivery cost on the product. The delivered price under these pricing policies is given by

$p\_f(x) = m_f + rx$ f.o.b mill pricing,

$p_u(x) = m_u = \text{constant}$ uniform pricing,

$p_d(x) = m(x) + rx$ optimal discriminatory pricing.

The firm is a spatial monopolist. The x and (s - x) denote the distance of its factory from markets 1 and 2, respectively. The firm faces the linear demand function at each market as follows;

$$q_1 = \alpha - \beta p_f = \alpha - \beta(m_f + rx) \quad (1)$$

$$q_1 = \alpha - \beta p_u = \alpha - \beta m_u \quad (2)$$

$$q_1 = \alpha - \beta p_d = \alpha - \beta(m_1 + rx) \quad (3)$$

$$q_2 = a = bp_f = a - b(m_f + r(s - x)) \quad (4)$$

$$q_2 = a = bp_u = a - bm_u \quad (5)$$

$$q_2 = a = bp_d = a - b(m_2 + r(s - x)) \quad (6)$$

Where pi, mi and qi are delivered price, mill price and sales in market i (i = 1, 2) respectively, r is the constant transport rate, and a, b, a, and b are all positive constants.

If the monopolist adopts f.o.b. mill pricing policy, then the profit function is given by:

$$\pi^f(m, x) = \big(\alpha - \beta(m_f + rx)\big)(m_f - c) + \big(a - b\big(m_f + r(s - x)\big)\big)(m_f - c) \quad (7)$$

where c denotes the constant marginal cost. Solving the maximizing problem of (7) yields a monopolistic price and quantity at each market, under f.o.b. mill pricing monopoly as follows,

$$m^f = \frac{(\alpha-a)-\beta rx-br(s-x)+(\beta+b)c}{2(\beta+b)} \quad (8)$$

$$q_1^f = \frac{(\alpha-a)\beta+2\alpha b-(\beta+b)\beta c-\beta^2 rx+\beta br(s-3x)}{2(\beta+b)} \quad (9)$$

and

$$q_2^f = \frac{2a\beta+(a-\alpha)b-b(\beta+b)c+\beta brx-br(2\beta+b)(s-x)}{2(\beta+b)} \quad (10)$$

The optimal location problem is solved as follows,

$$\frac{d\pi^f}{dx} = \frac{\partial \pi^f}{\partial x} = r(m^f - c)(b - \beta) \underset{<}{\overset{>}{=}} 0 \quad \text{as } \beta \underset{<}{\overset{>}{=}} b \quad (11)$$

and

$$\frac{d^2\pi^f}{dx^2} = \frac{r^2(b-\beta)^2}{2(\beta+b)} > 0. \quad (12)$$

Equation (12) is strictly convex with respect to x. This implies that any intermediate locations between the markets are strictly excluded in this simple location model. Therefore, we can get

$x^f = 0$ if b > b, and $x^f = s$ if b < b.

If a firm use uniform pricing, the profit function is

$$\pi^u(m, x) = (m_u - c - rx)(\alpha - \beta m_u) + (m_f - c - r(s - x)(a - bm_u) \quad (13)$$

Solving the maximizing problem of (13), we can get a monopolistic price and quantity at each market, under uniform pricing monopoly as follows,

$$m_u = \frac{(\alpha+a)+\beta rx+br(s-x)+(\beta+b)c}{2(\beta-b)}, \tag{14}$$

$$q_1^u = \frac{2\alpha(\beta+b)-\beta(\beta+b)c-\beta^3 rx-\beta(\alpha+a)-\beta br(s-x)}{2(\beta+b)}, \tag{15}$$

and

$$q_2^u = \frac{2a\beta+(a-\alpha)b-(\beta+b)bc-\beta brx-b^2 r(s-x)}{2(\beta+b)}. \tag{16}$$

According to the envelope theorem of comparative statics, we get $\frac{d\pi^u}{dx} = r(a - bm_u - \alpha + \beta m_u) = 0$ and $\frac{d^2\pi^u}{dx^2} = 0$. This equation says that a profit function is linear with respect to x. Therefore, optimal location of the firm is anywhere between either end.

Now, we consider the case of spatial price discrimination. A firm has a profit function as follows,

$$\pi^D(m_1, m_2, x) = \{\alpha - \beta(m_1 + rx)\}(m_1 - c) + \{a - b(m_2 + r(s - x))\}(m_2 - c). \tag{17}$$

Solving the maximizing problem of (17) yields optimal prices as follows,

$$m_1^D = \frac{\frac{\alpha}{\beta}-rx+c}{2}, \quad m_2^D = \frac{\frac{a}{b}-r(s-x)-c}{2}, \tag{18}$$

$$q_1^D = \frac{\alpha-\beta rx-\beta c}{2}, \text{ and } q = \frac{a-br(s-x)-bc}{2}. \tag{19}$$

The optimal location problem is solved as follows,

$$\frac{d\pi^D}{dx} = \frac{\partial\pi^D}{\partial x} = r\{b(m_2 - c) - \beta(m_1 - c)\} = 0 \tag{20}$$

and we have

$$\frac{d^2\pi^D(x)}{dx^2} = \frac{r^2}{2}(b + \beta) > 0. \tag{21}$$

This equation says that a profit function is strictly convex with respect to x. Therefore, optimal location of the firm is at either end.

3. Comparisons Between Alternative Spatial Monopoly Pricing Policies

We focus our attention on the spatial simple monopoly here. Hwang and Mai[1] assumed that under conditions of simple spatial monopoly the firm provides its product in both markets even if it can enjoy greater profit from only one of them, thereby abandoning the other market. To demonstrate our claim we adopt their particular parameter values of demand, cost, and transport rate structures. They are s = c = β= 1, a = α = 10 and b = 2.6. Using the equations of (3), (6), (7) and (11), we get

$$q_1^f = \frac{24.2-3.1r}{3.6} \text{ and } q_2^f = \frac{5.32-1.3r}{3.6}$$ [1]

The demand quantity under simple monopoly must be positive for each market. This requires $r < \frac{24.2}{3.1} = 7.8064516 \wedge r < \frac{5.32}{1.3} = 4.0923076$ and hence, r ∈ (0, 4.09) for both markets to be served. Thus, profit under simple monopoly is given by,

[1] These demand functions are derived from the aforementioned parameters and *x = 1* since β < *b*.

$$\pi^{f}_{1+2} = \frac{(8.2-0.5r)(29.52-1.8r)}{3.6^2} \qquad \forall r \in (0,4.09). \tag{22}$$

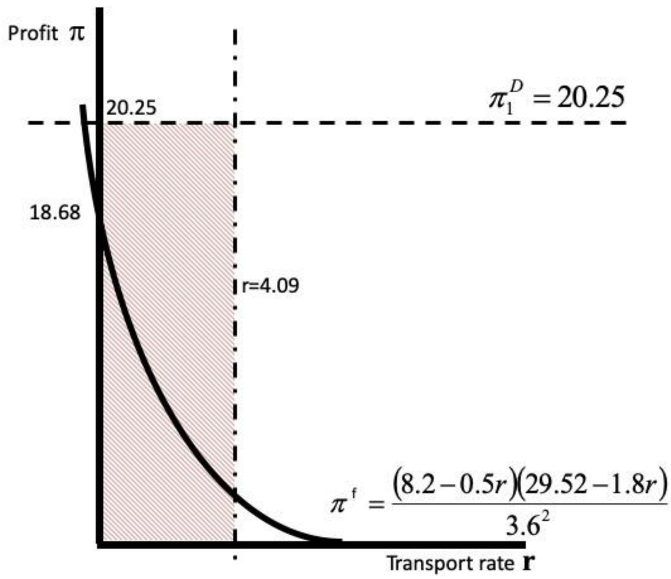

Figure 1 Profit Comparison Between the Simple Monopoly Pricing Policy With Selling at Both Markets and the Spatial Discriminatory With Selling at Single Market

Under simple monopoly suppose the firm choose to limit its sales to either one of the two markets, not both. Then it must locate at market 1, charge m = 5.5, and the sales quantity at that market alone must be q1f = 4.5. Therefore, if the firm sells their products only at market 1, we have the profit function as follows:

$$\pi^{f}_{1} = (m-1)q^{f}_{1} = 20.25 \tag{23}$$

Equations (21) and (22) are illustrated by Figure 1. Equation (21) is a decreasing function of transport rate. This shows how profit decreases as r increases for the simple monopoly making sales at both markets. Over the freight rate domain between r = 0 and r = 4.09, the Figure displays the profit from selling at a single market being 20.25 always greater than that from multi-markets under spatial simple monopoly.

A note is warranted here. With Hwang and Mai's parameters, a profit maximizing firm subjected to non-discriminatory pricing is not willing to supply the poor market 2 unless forced to, but instead if sells at the rich market 1 only.

4. Comparisons Between Spatial Non-discriminatory Monopoly and Discriminatory Monopoly

Next, we will consider the spatial price discrimination policy with Hwang and Mai[1]' s parameters as follows;

$q^{d}_{1} = \frac{9}{2}$, $q^{d}_{2} = 3.7 - 1.3r$, $m^{d}_{1} = \frac{11}{2}$, and $m^{d}_{2} = \frac{3.7}{2.6} - \frac{r}{2}$, $\forall r \in (0, 2.846)$.

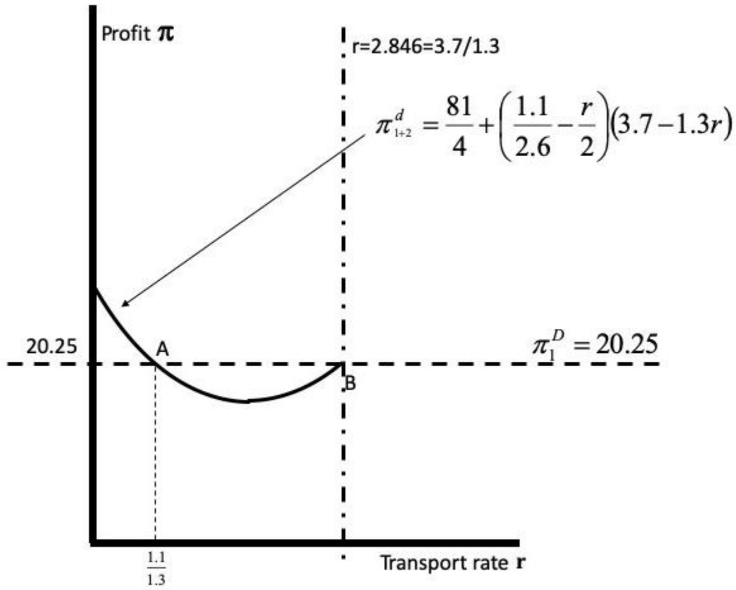

Figure 2 Profit Comparison in the Spatial Price Discrimination Policy Between Selling at Both Markets and Selling at Single Market.

Then, we have a profit function, in terms of transportation rate, as follows:

$$\pi^d = \frac{81}{4} + \left(\frac{1.1}{2.6} - \frac{r}{2}\right)(3.7 - 1.3r) \qquad \forall r \in (0, 2.846) \tag{24}$$

See the above page of Figure 2, the profit function under the spatial discriminatory pricing is illustrated as a convex downward curve. An effective domain of r is defined between zero and $r = \frac{3.7}{1.3} = 2.846$. A vertical dashed line shows one end of the effective domain of r. A horizontal dashed line represents the profit function under the simple monopoly pricing policy that optimally excludes the market 2. A spatial monopolistic firm must choose discriminatory pricing or any transportation rate between zero and $\frac{1.1}{1.3}$. If the transportation rate is sufficiently large (say, between A and B, or $r \in \left(\frac{1.1}{1.3}, \frac{3.7}{1.3}\right)$, Figure 2), the firm has to select the simple monopoly pricing. Furthermore, it must sell only in the rich market 1.

Finally, we compare the welfare effects under the two pricing policies. We follow Schmatensee to do this. Using the demand functions (3) and (6), we can obtain the consumers 'surplus at each market as follows;

$$CS_1 = \frac{1}{2}\left(\frac{\alpha}{\beta} - p_1\right) q_1 = \frac{q_1^2}{2\beta} \tag{25}$$

$$CS_2 = \frac{1}{2}\left(\frac{a}{b} - p_2\right) q_2 = \frac{q_2^2}{2b} \tag{26}$$

Based upon Equations (25), (26), and the profit functions which we obtained above, we can derive the welfare function with Hwang and Mai [1] 's parameters under the spatial price discrimination policy as follows;

$$\begin{aligned} W^d &= \frac{\left(q_1^d\right)^2}{2} + \frac{\left(q_2^d\right)^2}{5.2} + \pi^d \\ &= \frac{81}{8} + \frac{(3.7-1.3r)^2}{5.2} + \frac{81}{4} + \left(\frac{1.1}{2.6} - \frac{r}{2}\right)(3.7 - 1.3r) \\ &= \frac{3^5}{2^3} + \frac{(3.7-1.3r)(5.9-3.9r)}{5.2} \qquad \forall r \in (0, 2.846) \end{aligned} \tag{27}$$

And also, we have a welfare function under the simple monopoly pricing policy if the government forces the firm on selling at both markets as follows,

$$\mathrm{W}_{1+2}^{\mathrm{f}} = \frac{(24.2-3.1)^2}{2\times 3.6^2} + \frac{(5.32-1.3)^2}{5.2\times 3.6^2} + \frac{(8.2-0.5r)(29.52-1.8r)}{3.6^2} \qquad \forall r \in (0, 4.09). \tag{28}$$

If the firm sells only at market 1 under simple monopoly, the welfare function shall be given by:

$$\mathrm{W}_1^{\mathrm{f}} = \mathrm{CS}_1 + \pi_1 = 10.125 + 20.25 = 30.375 \tag{29}$$

Figure 3 shows the relationships among these 3 policies. The horizontal heavy line, $\mathrm{W}_1^{\mathrm{f}} = 30.375$, represents the spatial simple monopolist selling at the market 1 only. The dashed curve sloping downward, $\mathrm{W}_{1+2}^{\mathrm{f}}$, represents spatial simple monopoly selling at the both markets. An effective domain of r is also defined between zero and $\frac{3.7}{1.3}$. It is obvious that there is no incentive for the firm to provide all the markets since selling at market 1 only while ignoring market 2 is more profitable for nay transportation rate. The convex curve, Wd, displays the case for spatial price discrimination.

The welfare level depends upon the relationship between the transportation rate and the three pricing policies. It is important to note that spatial price discrimination may not yield greater welfare than other policies. We could obtain the highest welfare level under the spatial it can be regulated not to practice discriminatory pricing. On the one hand if the transportation rate is between zero and $r = \frac{5.9}{3.9}$, the monopolistic firm will serve both markets. On the other hands if it is between $r = \frac{5.9}{3.9}$ and $r = \frac{3.7}{1.3}$, the firm sells only at market 1.

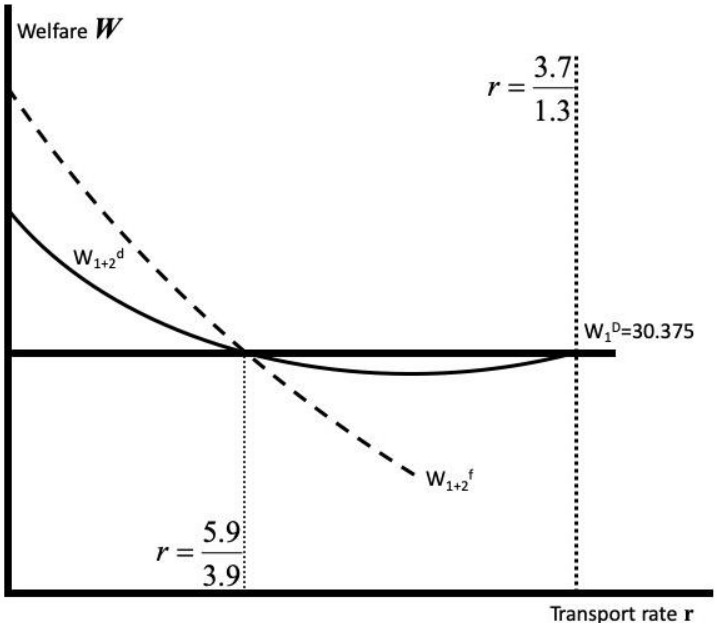

Figure 3 Welfare Comparison Among 3 Pricing Policies

5. Policy Implications

It is thus impossible to achieve welfare maximization without any government intervention or first-degree price discrimination within the confines of the parameters set provided by the Hwang-Mai simulation intended to show superiority of the third-degree discriminatory pricing over non-discriminatory pricing. The welfare maximizing pricing is not equal to the profit maximizing pricing. In an anarchy world, a monopolist would adopt the spatial price discrimination policy if the transportation rate remained low enough, say between zero and $\frac{1.1}{1.3}$, but it would choose simple monopoly pricing and sell at market 1 only if the transportation rate become high enough to exceed $\frac{1.1}{1.3}$, while remaining below $\frac{3.7}{1.3}$. If freight rate become even higher than $\frac{3.7}{1.3}$, no government regulation would indeed be warranted. For over the range of transport rate between $\frac{5.9}{3.9}$, and $\frac{3.7}{1.3}$, a monopolist would adopt simple monopoly pricing selling at a single market at its own initiative. Hence between $\frac{5.9}{3.9}$, and $\frac{3.7}{1.3}$, of the transportation rate, the welfare maximizing pricing is equal to the profit maximizing pricing. However, the rate between zero and $\frac{5.9}{3.9}$, the government can subsidize the simple monopolist to serve at both markets in order to achieve the maximizing welfare level.

6. Conclusion

The Hwang and Mai [1990] analysis of spatial monopoly pricing and location and related welfare comparisons is to non-discriminatory pricing in terms of welfare. Their assumption a firm's providing both markets even if the firm does not have any incentive to sell both markets under the simple monopoly. Under simple monopoly pricing the firm avoids selling both markets since it makes small profit, we can see in the Figure 1. Without that their assumption the spatial price discrimination does not require from a policy maker point of view. If a government imposed simple monopoly pricing upon a monopolist, the firm might sell both markets when the transportation rate is low, or, it might take on the simple monopoly pricing with providing single market when the transportation rate is high. The changing point is $r = \frac{5.9}{3.9}$ in their parameters.

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