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The "Circle" and the "Philosophy" of Geometry: Teaching Proposal for Students of Second Class of Senior High School

Thanasoulia Georgia[1], Gakis Panagiotis[1], Afratis Georgios[2]
(1. Modular High School of Patras, University of Peloponese, Greece; 2. Secondary Education of Achaia, Greece)

Abstract: This presentation deals with the presentation of a didactic proposal addressed to students of the second class of senior high school. The aim of this educational activity is to actively engage the student, in order to shape a "researcher" attitude, taking advantage of the possibilities of supervision and experimentation provided by a computer. In addition, an innovative aspect of cognitive reality arises, specifically interdisciplinary teaching. As far as philosophy is concerned, students realize the relationship between philosophy and mathematics, as well as the geometric triangulation that dominates the daily life of ancient Greece, art and poetry (in this case, comedy). Students realize the presence of mathematical rules in their daily lives. Students discover and ascertain the relationship of the perimeters of inscribed and delineated regular polygons to the length of the circle circumference. Geogebra software was used to implement this activity, a free and multi-platform application in the teaching of mathematics at all levels of education, which incorporates Geometry, Algebra, tables, graphs and Calculus. This paper presents the theoretical framework for planning and implementing the activity, as well as the mathematical approach to the problem and finally the presentation of the teaching process in classroom conditions.

Key words: number π, "geometry" of thoughts

1. Introduction

The modern post-industrial era is distinguished for the evolution of social coexistence, the increase of needs and demands, the accelerating pace of technology and, more generally, for the complexity of life. One of the key characteristics of today's society that holds a leading position in the organization of advanced countries is specialization. Through it, someone expects to follow the pace of development of technology and to cope with the demands of modern culture. Specialization is considered a necessary and inevitable process of life for the period of the industrial revolution up to the present day. Such an approach, however, leads to spiritual one-sidedness and "debilitates" human sensitivity and imagination. It also reduces a person's receptivity, narrows his spiritual horizons, weakens his critical ability, and leads him to a lack of self-expression.

The model that dominates our educational system is based mainly on the independent teaching of the various

Georgia Thanasoulia, Med, Modular High School of Patras, University of Peloponese; research areas/interests: mathematics. E-mail: geothanas@sch.gr.
Panagiotis Gakis, Ph.D., Med, Modular High School of Patras, University of Peloponese; research areas/interests: computational linguistic. E-mail: gakis@sch.gr.
Georgios Afratis, Ph.D., Med, Secondary Education of Achaia; research areas/interests: mathematics. E-mail: gafratis@gmail.com.

subjects, a model that does not ensure the required “internal coherence” and the “uniform horizontal development of their contents”. The Curricula (APS) should be connected horizontally in a way that ensures the multi-aspect approach of the various topics, in order to be “illuminated in many ways” and to highlight the knowledge they contain and its relationship with reality.

Modern pedagogical proposals are updated based on the requirements and problems of society. Hence a new teaching method comes to be the innovation of cognitive reality: the interdisciplinary approach of knowledge. The term “Interdisciplinarity” indicates the need for the student to examine more easily the “subject to be learned” object (topic) and to ensure his different point of view and different approaches (Theodoropoulos, 2004). More specifically, the “interdisciplinary approach” is defined as the multifaceted investigation and study of a topic that touches on many disciplines (Theodoropoulos, 2004).

New pedagogical and didactic methods based on interdisciplinary collaboration and the interdisciplinary approach of knowledge considers experimentation and exploration in the learning process necessary. In this way, they expect that traditional beliefs will be overcome and "watered down" and at the same time the development of a new way of thinking required by the new scientific and social developments will be supported. In this case learning does not mainly mean the transmission of knowledge, but is the result of the interaction of events in the human environment. The school is transformed into a student-centered, socio-centered and experiential, with all its participants, an attractive space and not only a space of standard teaching, a space for cultivating the student’s creativity (Alachiotis, 2002). The course with this approach can be carried out either in the context of the separate courses of the school curriculum or integrated into an interdisciplinary, detailed curriculum (Matsagouras, 2002).

The interdisciplinary approach has its roots in both Progressive Education, which flourished in the United States. In the second half of the 20th century as well as in the Labor School that dominated in Europe during the interwar period and was based on pedagogical positions analyzed by the educators Rousseau, Pestalozzi, and Froebel (Tzani, 2004). In Greece, the term interdisciplinarity is found in the new curriculum of the Interdisciplinary Unified Curriculum Framework (DEPPS). DEPPS, maintaining the distinct courses, includes various ways of correlating knowledge in two axes, the vertical and the horizontal. The first corresponds to the aggressive designation “Single” and the second to the aggressive designation “Interdisciplinary”.

The theory of “knowledge construction” or “constructivism” (constructivism) is based on the developmental theory of the scientist Jean Piaget and is the most modern and accepted theory around learning and teaching (Toumasis M. 1992). Constructivism, as applied in the teaching of Mathematics, focused almost exclusively on the processes by which students will actively construct their own Mathematical reality (Cobb P., Wood T. & Yackel E., 1991). The mathematics teacher should, through properly designed teaching situations, enable the student to construct knowledge. This design should be based on active learning methods, stimulate the student’s interest and help him “visualize” the concepts and relationships under negotiation. This will activate the student to form a “researcher” attitude: to make assumptions and verify them using previous knowledge, to try to generalize the results of “tests”, to model. The Mathematics teacher, based on the heuristic methodology of Polya (1991). How to solve it) can divide the problem into simpler problems and through individual questions to create a learning environment through which the student can discover the structure of the problem and the relationship of the parties to the whole.

The development of computer technology motivates us to use it more and more in the educational process. Especially in the teaching of Mathematics, the use of computers allows students to better understand mathematical concepts, facilitates experimentation and exploration, creates positive attitudes towards Mathematics, strengthens

supervision and, through their “visualization”, enables the most effective teaching of some concepts (Toumasis, 2003). However, a balanced use of computers in teaching and learning Mathematics is necessary. Some “pencil and paper” symbol handling skills are important for the learning process and balance requires an appropriate dose of “mental” and modern technology on a daily basis. Technology offers us the opportunity to teach Mathematics in an non-conventional way, reshaping its content and pedagogical dimension, giving it a new lease of life and not using it is unjustifiable (Toumasis, 2003). Geogebra belongs to the category of multi-representation softwares, enabling access to knowledge in an interesting way, including playful learning.

It is free dynamic software for teaching mathematics that combines geometry, algebra and calculus. There are two typical aspects of GeoGebra: An expression in the algebra window corresponds to an object in the geometry window and vice versa. So students can work in each of the windows and observe the results of their actions in the other.

The connection of geometry with philosophy and art was the basis for the design of the interdisciplinary approach of our course. On the occasion of the above concerns and driven by the interdisciplinary framework of studies, a lesson based on the previously analyzed concepts was designed by two teachers for a section of students of the 2nd grade of the Experimental Senior High School of Patras (25 students). The implementation of the activity was completed in two teaching hours and was conducted in the computer lab. The course conducted is that of collaborative - exploratory learning (group work of 2 or 3 people - guided discovery). The role of the teachers was advisory, guiding, supervisory, asking appropriate questions to the groups and if they deemed necessary helping groups that were behind and managing the time so that there was not a big difference between the processing stages of the different groups.

1.1 Necessity of Interdisciplinary Learning

The main points of our reflection (?) were the following:

1) Students’ “over-information” of various elements tests their mental structures and way of thinking. They therefore approach the fields of science in one dimension with obvious difficulty in processing, judging and understanding the sheer volume of information.
2) Modern scientific research requires the cooperation of scientific specialties that are seemingly characterized as “unrelated”. In fact, there are university departments that have included such collaborations in their research field (eg computational linguistics).
3) The hierarchy of taught courses and the specialization (direction-general education courses) leads to the perception that the taught objects operate in competition with each other, setting aside basic pedagogical functions.
4) The stimulation of students' self-confidence and self-esteem through their cooperation is an element that the modern school should seek.
5) Familiarity of students with conclusion formulation and critical thinking should characterize daily educational practice.

For all the above reasons, we consider it necessary to experiment with new pedagogical and didactic methods that will be based on interdisciplinary cooperation and the interdisciplinary approach to knowledge. In this way we believe that the chances of overcoming traditional perceptions and “watertight” increase.

1.2 Teaching Objectives

The design of interdisciplinary teaching aimed at the following objectives:

1.2.2 Cognitive objectives:

- To define, to formulate a problem from an extra-mathematical area with mathematical thinking,
- to discover relations between the Mathematics and other cognitive areas of the study programδών (?),
- to calculate the length of the circle using an approximation procedure,
- to know the special nature of the number π as well as its historical value

1.2.2 Skills level goals:

- the promotion of collaborative learning and communication skills,
- encouraging dialogue and communication.

1.2.3 Targets at the attitude level:

- the establishment of connections of seemingly “irrelevant” scientific specialties,
- the removal of the hierarchy of courses taught and the cancellation of their competition,
- stimulating their self-confidence and self-esteem through cooperation,
- familiarity with drawing conclusions and developing critical thinking towards “diverse” information.

2. Teaching Course

This course is inspired by the science of philosophy. Students are first asked about the relationship between philosophy and mathematics (and especially geometry). With a brief review, the interdependence of the two terms is highlighted and their differences and similarities are analyzed.

Students then observe the geometric triangulation that dominates ancient monuments and cities. (eg distance of Delos from the Asclepieion of Kos and Epidaurus). Reference is also made to the painting of Raphael (Figure 1) and the hidden geometry that dominates it. The center of the table is the circle based on which the speech becomes clear “*Μηδείς* ἀγεωμέτρητος εἰσίτω”. Circle also dominates the Socratic “Ἓν οἶδα ὅτι οὐδὲν οἶδα” as well as in Aristophanes’ “Ornithes”, where Metonas tries to square the circle. Meto-nas appears to Peisetairos, founder of Nefelokokgygia, to help plan the new city, with the certainty that he knows how to… square the circle! Students are led to the dilemma: “*do we live in a world determined by fixed mathematical rules that man discovered and recorded or is mathematics a human construct that supports our creations*” Is it knowledge that legislates and creates the reality we experience?

The mathematics teacher utilizes these classroom concerns in the Geogebra educational software environment and students with appropriate worksheets are led to discover the relationship of the perimeters of inscribed and delineated regular polygons to the circumference of the circle. This discovery is expected to stimulate the interest of every student in research and lead to the discovery of new avenues for the study of objects and concepts.

Figure 1 The School of Athens

3. Description of the Teaching Steps

Step 1: Students are asked to think about the relationship between philosophy and mathematics. In the interactive discussion that takes place, the Socratic saying is analyzed in a geometric shape: “*Ἓν οἶδα ὅτι οὐδὲν οἶδα*”. Students associate the Socratic saying with the geometric shape of the circle. More specifically, what is contained within the circle is the knowledge that one possesses and the knowledge that hasn’t yet possessed lies outside of the circle. Consequently, the larger the area of the circle, the more one realizes the magnitude of the knowledge he does not know. With this interface students are expected to better understand the depth of the philosopher’s thought.

Step 2: Students observe the geometric triangle that dominated ancient monuments and cities. The students observe the maps that are displayed and their teacher points out this geometric triangulation (for example that Elefsina is 100 steps away from Athens as well as from Megara). Students are expected to reflect on the fact that the locations of cities, temples and places of worship seem to be for some unknown reason calculated, with mathematical systems! The interactive discussion that takes place does not aim at formulating an answer, but at enhancing the students’ reflection and discovering the geometry behind culture.

Step 3: The students observe the painting “Rafaelos School of Athens”. This painting depicts the famous philosophical School of Athens in antiquity, in which philosophers, scientists and poets of antiquity seek the truth through dialogue. With the guidance of the teachers, they connect the faces of the board with geometric shapes (triangles, bisectors of angles, circle) and discover the hidden geometry of the board and consequently the dominance of the saying: “Μηδείς ἀγεωμέτρητος εἰσίτω” in the art. Students are expected to observe a general semicircular definition that dominates the table (centered on Plato and Aristotle), and their wonderful geometric connection (and therefore philosophy) with historiography, mathematics, rhetoric, speech and work (Figure 2).

Step 4: The students read an excerpt from Aristophanes’ Ornithes (Aristophanes, Ornithes, pp. 992–1009) in which the protagonists tried to square the circle. They work in groups and plan a theatrical presentation of the excerpt. The teacher invites one or two groups to represent to their classmates the dramatized dialogue of the relevant text using the guide and diabetes.

Figure 2 The School of Athens

Step 5: We ask the following question: “if r is the radius of a circle what is the formula you know for calculating the length of its circumference?”

Step 6th: Students are asked to open the geogebra file on the PC the .ggb circle length (Students) (Figure 3) on their PC and give appropriate values to the cursor n, to construct inscribed and outlined regular polygons with number of sides of your choice but gradually increases (e.g., 4, 6, 8, 12, 24, 48,…). Students are then asked to observe which of the calculated quantities change, which remain constant and to complete the following tables with the figures of the problem.

To shorten the work, the students are divided into groups and each group is assigned to complete one or more lines in Tables 1 and 2.

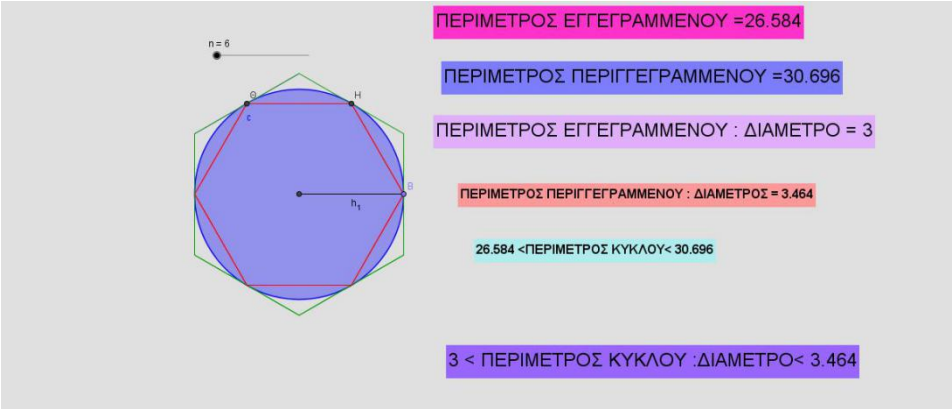

Figure 3 Circle Length .ggb (Students)

Table 1 Table With the Figures of the Problem

Registered

Number of Sides	Perimeter (Π)	Cycle length	Diameter (D)	Π/D
6				
12				
24				
48				
96				

Table 2 Table With the Figures of the Problem

Outlined

Number of Sides	Perimeter (Π)	Cycle length	Diameter (D)	Π′/D
6				
12				
24				
48				
96				

Step 7: Students are asked to identify the relationship between the perimeters of the inscribed, the inscribed regular polygons and the length of the circumference of the circle. Finally, students are asked to conclude empirically what is the relationship between the length of the circle L and the length of the diameter of the circle D.

Step 8: With the help of a ready-made file in the environment of the Geogebra application, the conclusions are checked and interpreted. The program enables the student to see the dynamic change of polygon perimeter and circle length sizes and the numerical approximation of the circle length from the value of the perimeters of the inscribed and outlined polygons as the number of sides increases.

The proposed educational lesson plan is not only an innovation in the traditional context of the teaching of this unit of Mathematics but also is expected to have a wider influence. Specifically, it allows an experiential connection of the algebraic relation and the geometric representation of the above concepts and aspires to contribute to the improvement of students' attitude towards Mathematics and the way students approach this lesson, but also to the realization that Mathematics are explorable. In fact, each student can test in this context his own ideas and come to his own conclusions which must have the appropriate social acceptance (in the classroom) and of course should be based on proper scientific documentation. The use of technological tools is expected to facilitate significantly in this direction. At the same time, students are expected to be led to connections of both mathematics with the cultural environment in which they are produced and of the preservation and progress of culture with the level of development of mathematics.

Step 9: Students are given tasks such as forming music with the number π or forming a poem with words that have an equal number of syllables with the number π. Both the music and the poem are created in relation to its decimal part and each decimal digit of π).

Step 10: In this step the results are generalized. Here is an interesting twist: The teacher presents a brief historical overview of the number π, Archimedes' method of exhaustion for its calculation as well as the continuous effort of mathematics worldwide to this day to calculate more and more of its digits.

The evaluation of the above teaching can be conducted in three axes:

- **the formative assessment** that will be conducted during the teaching through the observation and the interest of the students, through their questions and more generally through the observation of their work in the group,
- **the use of web addresses and software** in order for students to gather information, produce material and present it to their classmates,
- **the final assessment** using questionnaires that will be given to both students who participated in the teaching and the teachers who attended it.

4. Conclusions

Closing this brief description of this innovative action and making a final assessment of its application, it could undoubtedly be concluded that the interdisciplinary approach to knowledge is an attractive way of teaching for both students and teachers. We believe that interdisciplinary teaching influences students’ way of thinking and allows them to express themselves with greater ease and flexibility. In addition, the cognitive background that students were able to acquire was high, as the horizons of mathematical thinking were broadened because they were given the opportunity to establish connections between mathematics and philosophy and therefore human thought. Also, the students who attended theoretical direction became more receptive to the approach of mathematical thinking, realizing that mathematics is in direct correlation with philosophy and logic.

For the teachers, this way of teaching was a new experience that they really enjoyed through sharing the common love of Mathematics and Philosophy with their students and the interaction that came from it. Such teaching places students at the center of the teaching process, respects and supports creative communication between all participants and expands the possibilities of their personal expression.

Based on the answers given to the questionnaires, it was observed that the evaluation of this teaching by students and teachers is approached from different perspectives and with different objectives, depending on the particular interests of the evaluation teams.

Therefore, the students almost unanimously judged that the teaching stimulated their interest more than the “formal” lesson. They found the motivation given by the philologist attractive and considered the fact that they saw another learning context as an advantage of the teaching. The role of audiovisual material was assessed as equally important.

For teachers, the measure and criterion for the evaluation of teaching were the goals that were initially set and which in their view were fully met. They observed the students’ actions in the activities they participated in and felt that they understood the meanings of the lesson. At the same time, it was estimated that the interdisciplinarity of teaching helped to develop an effort to acquire knowledge from different perspectives, with the result that students of theory approach more easily, in relation to traditional teaching, mathematical concepts and students of positivity philosophical propositions.

This image was reflected in the discussion that followed after the lesson between those who attended the teaching. The discussion was led by the competent school counselor of Mathematics who also organized the monitoring of the teaching. The teaching was attended by the school counselor of philologists, the principal of the school where it took place, branches of the above school, as well as teachers of corresponding specialties from two other neighboring schools. Teachers commented on the experiential element of teaching (knowledge discovery, science interconnection, pleasant classroom atmosphere) while noting that teaching was more responsive than traditional teaching both in terms of teaching effectiveness (e.g., whether students learned effectively) as well as the degree or pace of student response (whether student needs are met and how quickly this happens). During the discussion there were questions about the students’ grades in these as well as in other subjects and it became clear that the success of teaching does not depend on the high performance of students in mathematics or philosophy, since the teaching involved a group of students with significant differences. in their performance. From the exchange of views and positions it emerged that the success of teaching allows its continuation and expansion in the field of philosophy of mathematics or even in the interconnection of mathematics and literature.

References

Alachiotis S. (2002). “For a modern educational system”,*Educational Issues Inspection*, Vol. 7, pp. 7–18.

Argyropoulos I., Vlamos P., Katsoulis G., Markatis S. and Sideris P. (2012).*Euclidean Geometry A and B General Lyceum*, ITYE ΄DIOFANTOS΄.

Matsagouras I. (2002). “Interdisciplinarity, interdisciplinarity and unification in the new curricula: Ways of organizing school knowledge”,*Educational Issues Inspection*, Vol. 7, pp. 19–36.

Polya G. (1991). *How Will I Solve It*, Athens: Kardamitsa.

Theodoropoulos P. (2004).“The interdisciplinary approach to knowledge: From philosophy to application”.

Tzani M. (2004). *Interdisciplinarity in the All-Day Pilot School*, Athens.

Toumasis M. (1992). *Contemporary Didactics of Mathematics*, Athens: Gutenberg.

Toumasis M. (2003). *Teaching Mathematics Using A Computer*, Athens: Savala.

Available online at: http://users.sch.gr/atmatzidis/aa_pps/Sxoli_ton_Athinon_tou_Raffaello.pps.

Available online at: http://www.geogebra.org: GeoGebra is dynamic math software for all levels of education.

ANNEX

Interdisciplinary teaching evaluation form for teachers

We want your opinion to continue to improve the course! Fill out the form below to tell us where points need improvement and which points are correct and should remain.

1. Did interdisciplinary teaching - as organized - stimulate students' interest more than a 'formal' lesson?

YES…… NO……

2. The aims of the lesson coincide with what was taught in the classroom?

YES…… NO……

3. Were the classroom activities well planned? What would you like to observe (positive or negative)?

YES…… NO……

4. The motivation given to the students by the philologist (if yes, in which field) helped;

5. What do you think is the most important advantage of teaching?

What do you have to suggest in order to improve this teaching method? What do you, in your opinion, think was the most important disadvantage of teaching;