Modern Environmental Science and Engineering (ISSN 2333-2581)
June 2020, Volume 6, No. 6, pp. 708-714
Doi: 10.15341/mese(2333-2581)/06.06.2020/009
Academic Star Publishing Company, 2020
www.academicstar.us

Voltage Control, By Reactive Power Injection

Johonel Caceres Espinoza[1], and Lucio Caceres Espinoza[2]

1. National University of San Antonio Abad of Cusco, Peru

2. National Technological University of Lima South, Peru

Abstract: The growth of electric power systems due to the increase in demand, generates the need to have a greater number of interconnections between the various generation - load systems, because they allow to serve users with a lower operating cost, attend Sudden load variations, purchase and sale of energy, but these increases, cause a decrease in the margins of chargeability and decrease in the voltage levels in the bars of a power system, resulting in the risk of bringing the system to Tension instability. These decreases can also be produced by the limited injection of reactive power, lack of use of compensation elements, etc. An important aspect of the stability of tension is that it is a non-linear phenomenon, which is why computational methods and tools are used to control the tension in the bars, by means of the injection of reactive power. Therefore, a computational tool that expands the Jacobian matrix will be implemented in order to increase a new state variable, within the Jacobian matrix, which helps to improve tension levels in the face of increased demand

Key words: power flow, modal analysis, sensitivity matrix

1. Introduction

The growth of Electric Power Systems due to increases in demand, gives the need to have systems that offer greater reliability and lower operating costs, so it is necessary, increasing interconnections, in order to satisfy the demand.

Under this context, interconnected systems are advantageous, because they allow electric power generation companies to meet the demand, at peak hours, with a smaller number of generating machines that operate in a vacuum (rotating reserve), with the purpose of attending sudden loads, purchase and sale of energy, etc. And have a better use of the resources used for the generation of electrical energy (hydraulic, thermal, combined cycle, etc.).

These interconnections also generate some inconveniences in practice, such as an increase in the sizing of the systems and a very complex coordination of operation, causing short-circuit currents to increase significantly. From this point of view, we observe that a detailed planning of system operation is necessary for its performance to be compatible with market requirements. It also becomes necessary for its operation, to have a detailed knowledge of its protection and to have tools that facilitate a quick analysis of its conditions of electromechanical stability and safety of tension in permanent regime.

Within this context, the calculation of power flow, constitutes the basis for the solution of various problems, referring to the operations of electrical power systems (short circuit analysis, reliability, optimal operations, Hydrothermal dispatch, stationary stability, etc.).

To perform the calculation of power flow, it is necessary to use numerical methods, due to the existence of non-linear equations present in the calculation of active and reactive powers, which are present in the transmission elements (transmission lines, transformers, etc.).

Corresponding author: Johonel Caceres Espinoza, Master in Electrical Power Systems; research area/interest: electrical power systems. E-mail: ycacerese1@gmail.com.

Currently, various power flow calculation programs use various variations of the Newton Raphson method [1-3].

In spite of requiring more complex algorithms for its implementation, Newton Raphson's method presents a quite satisfactory computational performance.

A very important feature of this method is its numerical robustness, as well as a reduced number of iterations, which allow its use for difficult-to-solve systems.

For this method to be appropriate, for the calculation of power flow with voltage control, it is necessary to introduce mathematical models, which allow to represent control devices, whose main function is to present electrical systems, more in line with reality, in order to improve the tension levels (tension module) in the bars and thus be able to avoid voltage collapses and that the system works within the safety margins (stability region).

2. Materials and Methods

2.1 Power Flows

The calculation of power flow is the most basic analysis tool used to know the steady-state operation of an electric power system. It is defined as the solution process that provides the steady-state voltages in all the nodes that make up the electrical network under analysis and based on which the active and reactive power flows injected into terminals of each transmission element under the calculation are calculated assumption of known values of power generated and consumed.

The way to obtain the stationary operation point of a power grid based on an analysis of power flows, is determining that the generation power, the load power and the power that is exchanged through the transmission lines must add zero on each of the nodes of the network (this applies to active power and reactive power). This can be expressed mathematically by a group of equations:

$$\Delta P_i = P_{Gi} - P_{Li} - P_i = 0 \qquad (1)$$

$$\Delta Q_i = Q_{Gi} - Q_{Li} - Q_i = 0 \qquad (2)$$

Where:

P_{Gi} and Q_{Gi} they are the powers of injected by the generator connected in node i of the network.

P_{Li} and Q_{Li} they are the powers extracted by the load connected in node i of the network.

P_i and Q_i are the powers that flow through the transmission elements and are calculated according to Eqs. (3) and (4).

$$P_i = V_i^2 G_{ii} + V_i \sum_{j=1}^{n} V_j\left(G_{ij}\cos(\theta_i - \theta_j) \ldots \right.$$
$$\left. + B_{ij} sen(\theta_i - \theta_j) \right)$$

$$Q_i = -V_i^2 B_{ii} + V_i \sum_{j=1}^{n} V_j\left(G_{ij}\cos(\theta_i - \theta_j) \ldots \right.$$
$$\left. - B_{ij} sen(\theta_i - \theta_j) \right)$$

The subscripts *i*, *j* represents the connection nodes of the transmission element.

Due to the obvious nonlinearity of Eqs. (3) and (4) the use of solution methods is necessary, for the solution of the Power Flow problem. There are several methods among which Gauss-Seidell, Newton-Raphson, Quick Disengagement, etc. stand out.

The Newton Raphson method has been used as an efficient method in terms of its characteristics of convergence speed, accuracy and memory requirements.

In order to apply the Newton Raphson method, it is necessary to have a symmetric system of equations, that is, the same number of unknowns and equations. In each node of the network two variables are specified and according to those specified variables the node is classified according to Table 1.

Table 1 Units for magnetic properties.

Node Type	Specified Variables	Calculated Variables
Node Swing	IVI y θ	P y Q
Node PV	IVI y P	Q y θ
Node PQ	P y Q	IVI y θ

Once the appropriate variables of the Power Flow equations have been specified, it can be solved, linearizing around the initial conditions (P^0, Q^0), Where do you get:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}^k = \begin{bmatrix}\frac{d\boldsymbol{P}}{d\theta} & \frac{d\boldsymbol{P}}{dV}\\ \frac{d\boldsymbol{Q}}{d\theta} & \frac{d\boldsymbol{Q}}{dV}\end{bmatrix}^k \cdot \begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix}^k \tag{5}$$

It can also be represented as:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}^k = [J_{ac}]^k \cdot \begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix}^k \tag{6}$$

Where:

J_{ac} It is known as the Jacobian matrix and contains the partial derivatives of active and reactive power with respect to the magnitudes of voltages and angles.

k Represents the iteration of the solution process.

2.2 Shunt Device Representation in Power Flow Problems

A flexible representation of Shunt devices in Power Flows, is obtained, increasing the original system of power equations, reached from Eqs. 3 and 4, this representation corresponds to a form closer to reality. The generic way of linearizing this augmented system of equations is solved in each iteration of the Newton Raphson method, which is shown through Eq. (7)

$$\begin{bmatrix}\Delta\boldsymbol{P}\\ \Delta\boldsymbol{Q}\\ \Delta\boldsymbol{y}\end{bmatrix} = \begin{bmatrix}\frac{d\boldsymbol{P}}{d\theta} & \frac{d\boldsymbol{P}}{dV} & \frac{d\boldsymbol{P}}{dx}\\ \frac{d\boldsymbol{Q}}{d\theta} & \frac{d\boldsymbol{Q}}{dV} & \frac{d\boldsymbol{Q}}{dx}\\ \frac{d\boldsymbol{y}}{d\theta} & \frac{d\boldsymbol{y}}{dV} & \frac{d\boldsymbol{y}}{dx}\end{bmatrix} \cdot \begin{bmatrix}\Delta\theta\\ \Delta V\\ \Delta x\end{bmatrix} \tag{7}$$

In this equation the vector $\Delta\boldsymbol{y}$ represents the error vector of the additional equations that model the control equipment. The vector $\Delta\boldsymbol{x}$ It is formed by the state variables incorporated into the power flow problem, at the end of each iteration the state variables are updated as follows:

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \Delta\boldsymbol{x}^{(k)} \tag{8}$$

Eq. (7) can be conveniently considered as follows:

$$\begin{bmatrix}\Delta w\\ \Delta y\end{bmatrix} = \begin{bmatrix}J_{ac} & J_{wx}\\ J_{yu} & J_{yx}\end{bmatrix} \cdot \begin{bmatrix}\Delta u\\ \Delta x\end{bmatrix} \tag{9}$$

Where vectors Δv and Δu are given by:

$$[\Delta w] = \begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} \tag{10}$$

$$[\Delta u] = \begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix} \tag{11}$$

The matrix J_{ac}, in Eq. (9) it represents the Jacobian matrix of traditional formulation of Newton Raphson's method, J_{wx} contains those derived from the power equations in relation to the new problem state variables. The blocks J_{yu} y J_{yx} represents those derived from the equations that model shunt devices in relation to the original and additional state variables respectively. The matrix J It is called Expanded Jacobian Matrix

$$J = \begin{bmatrix}J_{ac} & J_{wx}\\ J_{yu} & J_{yx}\end{bmatrix} \tag{12}$$

This expanded formulation allows great ease of incorporation of Shunt devices, taking into account that the matrix J_{ac} is preserved.

2.3 Sensitivity Analysis

It is well known that the problems of voltage stability in power systems are associated with the behavior of reactive power-system voltage, so some techniques based on the analysis of operating conditions in stable (or static) state such as in Refs. [1, 4].

The expanded Jacobian matrix will be used, for the determination of the sensitivity which relates the behavior of the variations of:

- The power injected into the nodes and their voltages.
- Remote voltage control with the use of Shunt compensators.

It is also important to mention that an injection of reactive power by Shunt compensators improves the performance of the system during the operation, keeping the module of the voltage levels of the bars as close to their nominal value, thus reducing the current on transmission lines and therefore reducing losses and contributing to an improvement in the safety margin of the system.

These injections of reactive power, obtains a family of P-V curves (Fig. 1), expanding the load margin of the electric power system, so it is considered an alternative solution, within the technology of FACTS devices.

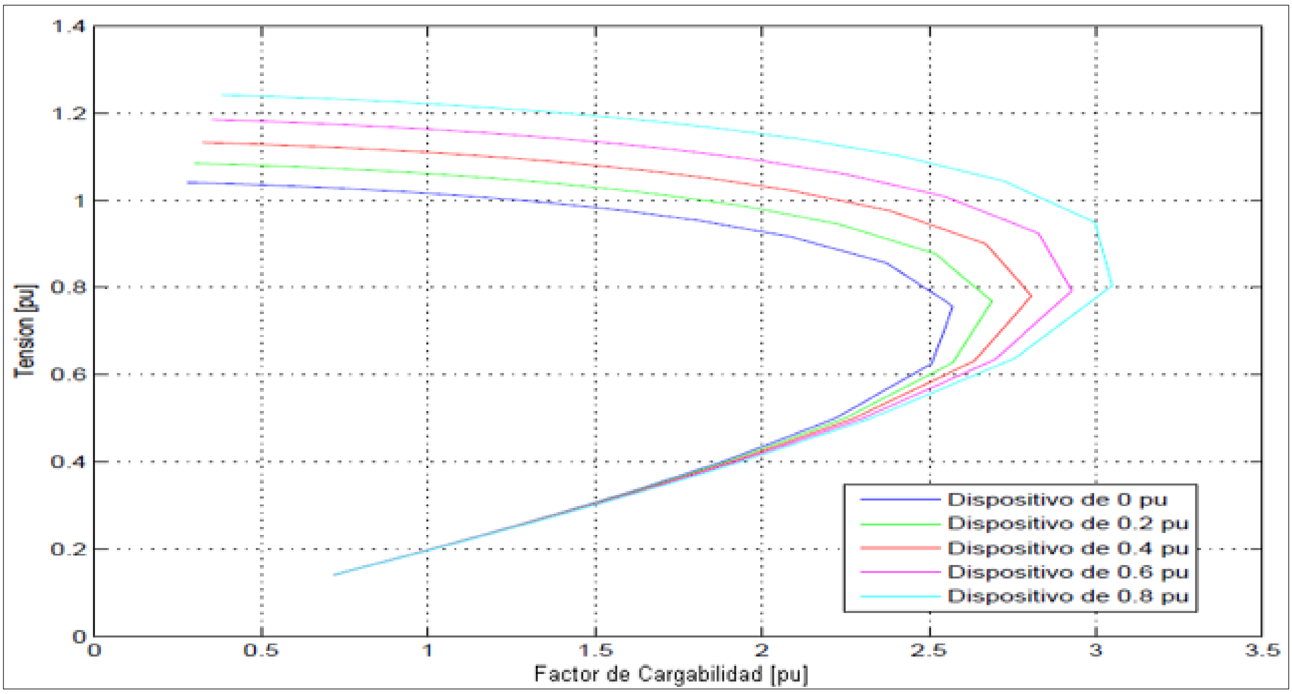

Fig. 1 Family of P-V Curves, by injection of reactive power.

2.4 Sensitivity QV

The QV sensitivity analysis will be determined from the matrix "J_{ac}" which represents the traditional Jacobian matrix. So, making $\Delta P = 0$, an expression is reached between ΔQ y ΔV.

$$\begin{bmatrix} 0 \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix} . \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} \quad (13)$$

$$\Delta Q = \left[-J_{Q\theta} * J_{P\theta}^{-1} * J_{PV} + J_{QV}\right] \Delta V \quad (14)$$

$$\Delta Q = J_R \, \Delta V \qquad y \qquad \Delta V = J_R^{-1} \, \Delta Q \quad (15)$$

Where:

$J_R = \left[-J_{Q\theta} * J_{P\theta}^{-1} * J_{PV} + J_{QV}\right]$, represents a Reduced Jacobian matrix.

As observed J_R is the matrix that directly relates the variations in magnitude of the voltage with the variations in magnitude of the reactive power injected into each node (gradients).

2.5 Values Analysis and Own Vectors

The modal analysis corresponds to the analysis of values and vectors characteristic of the sensitivity matrix J_R and J_{SC}. In this way the particular behavior of each sensitivity mode is characterized and through the vector analysis the influence of reactive load injections on certain nodes can be determined, whose analysis is known as nodal participation factors.

From this method, modal sensitivities are determined, obtained from the diagonalization process of J_R and J_{SC}, as follows

Para la Matriz J_R

$$J_R = \xi * \Lambda * \eta \quad (16)$$

Where:

ξ = Own vector matrices rights of J_R

η = Left own vector matrices of J_R

Λ = Diagonal matrices of own values of J_R

Now the voltage increases in the nodes are given by:

$$\Delta V = \xi * \Lambda^{-1} * \eta \; \Delta Q = \sum_{i=1}^{m} \frac{\xi_i * \eta_i}{\lambda_i} \Delta Q \quad (17)$$

Being ξ_i the i – esima column of ξ, η_i the i – esima row of η, and λ_i is the row and column i of Λ.

Since $\xi^{-1} = \eta$, the equation 17 can be written in the form:

$$\eta * \Delta V = \Lambda^{-1} * \eta \ \Delta Q \quad \Rightarrow \quad \mathbb{v} = \Lambda^{-1} * \mathbb{q} \qquad (18)$$

Where $\mathbb{v}$ and $\mathbb{q}$ are the modal variables of voltage and reactive power, respectively related:

$$\mathbb{v}_i = \frac{\mathbb{q}_i}{\lambda_i} \qquad (19)$$

For the Matrix J_{SC}

$$J_{SC} = \xi' * \Lambda' * \eta' \qquad (20)$$

Where:

ξ' = Own vector matrices rights of J_{SC}

η'= Left own vector matrices of J_{SC}

Λ'= Diagonal matrices of own values of J_{SC}

Now the voltage increases in the nodes is given by:

$$\Delta V = \xi' * \Lambda' * \eta' \ \Delta b_{sh} \qquad (21)$$

From equation 21, equation 22 is achieved

$$\eta' * \Delta V = \Lambda' * \eta' \ \Delta b_{sh} \quad , \quad \mathbb{v}' = \Lambda' * \mathbb{b}_{sh} \qquad (22)$$

Where $\mathbb{v}'$ and $\mathbb{b}_{sh}$ they are the modal variables of voltage and of the shunt device.

In equilibrium conditions always the values λ_i they are positive; however, the most critical are those that approach the voltage stability frontier, that is when the modal sensitivity is reversed (negative); in other words, the critical modes are those that approach 0.

On the other hand, the relative importance of each node (k) in modal dynamics or sensitivity (i) is given by the nodal participation factor (p_{ki}) defined by:

$$p_{ki} = \xi_{ki} * \eta_{\ ki} \qquad (23)$$

Since the resulting participation factors are normalized values whose sum over all the nodes for a particular mode is equal to 1, the greater the value of the participation factor, the greater the influence of node k on the sensitivity of mode i.

2.6 Singular Decomposition Sensitivity Analysis

This method decomposes the Jacobian matrix into four sub-matrices, which represent the partial derivatives of the active and reactive powers according to the state variables, with a $\Delta P = 0$, because this error vector is less than the tolerance margin, in addition to the strong relationship between the reactive power and the voltage module of the power system bars.

$$\begin{bmatrix} 0 \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\theta} & 0 \\ 0 & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} \qquad (24)$$

This new Jacobian matrix can be decomposed as indicated in Eq. (25).

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = U\Sigma^{-1}V^T \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \qquad (25)$$

The greatest value of Σ^{-1} occurs for the inverse of the minimum singular value of the matrix. The following relation for the minimum singular value and the corresponding singular vectors, obtained from equation 25 is of study interest.

$$v_n^T \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = \sigma_n^{-1} u_n^T \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \qquad (26)$$

The inverse of the minimum singular value σ_n^{-1}, from the point of view of small disturbances, it indicates the largest change in state variables.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = u_n \qquad (27)$$

Where u_n it's the last column of U, so:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \sigma_n^{-1} v_n \qquad (28)$$

Where v_n it's the last column of V. From all of the above, the following interpretations can be made for the minimum singular value and the corresponding right and left singular vectors.

3. Results

To show the importance of the proposed methods, the 14-bar IEEE test system will be used for voltage control.

For this system, two capacitor banks were connected in bars 5 and 14 of the test system, in order to perform a local and remote control of the voltages in bars 4 and 14, at desired voltage levels of 1,014 pu and 1.0512 pu.

According to Fig. 3, it is observed that the voltage levels of all the bars increased (with respect to their magnitude) as well as the tensions in the control bars, at the desired values, due to the injection of reactive power, by Shunt devices placed on bars 5 and 14.

Due to the connection of the voltage control devices, it resulted in the increase of two new state variables and two new equations for the power flow solution, which are necessary requirements for obtaining the operating point of the system power, so the process proves to be effective because the number of iterations came to converge in a few iterations (four iterations), as shown in Fig. 4.

Thanks to the calculation of the eigenvalues and singular values we can determine, that those bars that have the minimum value (Fig. 5), are those bars that are closer to the instability of tension, being for our case of study the bars 11, 13 and 14 closest to instability because they have roots equal to 2.7012; 5,564 and 7,666, present in the reduced Jacobian matrix.

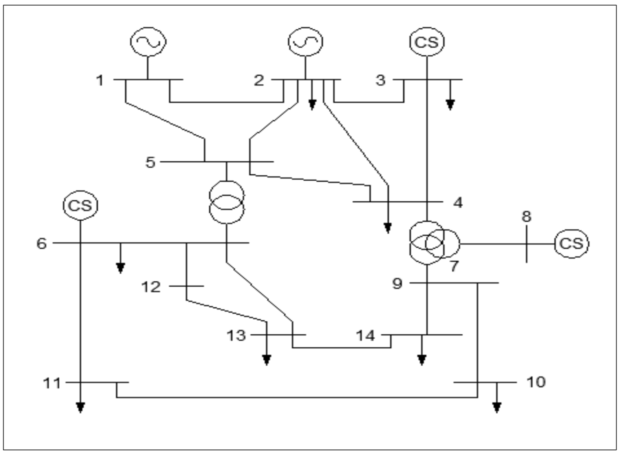

Fig. 2 Uniform diagram of the IEEE system - 14 bars.

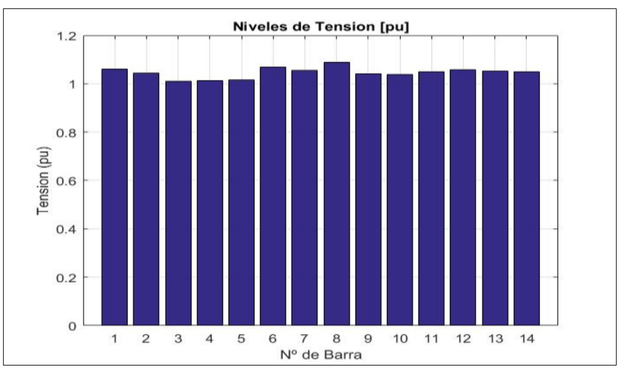

Fig. 3 Voltage levels on the bars, IEEE system - 14 bars.

Fig. 4 Number of iterations of the proposed method, for the IEEE system - 14 bars.

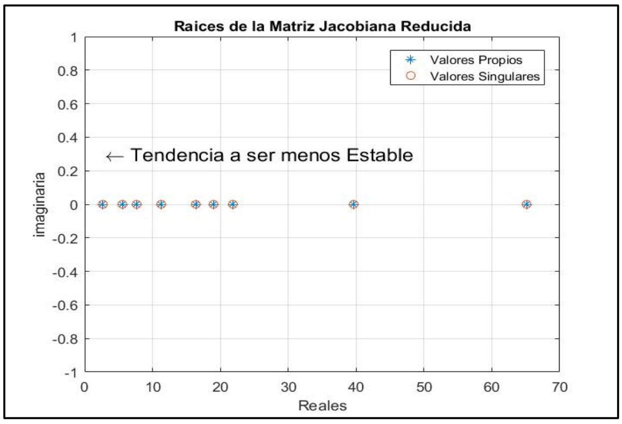

Fig. 5 Own values and singular values, of the IEEE system - 14 bars.

Another calculation, to determine those bars close to voltage instability, is by calculating the participation factors, obtained from the decomposition of the right and left vectors, for a minimum root (minimum own value and minimum singular value), which shows the most extreme operation of the system.

In addition, in Fig. 6, it shows that the bars closest to instability are presented in bars 14, 10, 9 and 11 which show that these bars are prone to reactive power injection, in order to remain in the stable region of the system.

From the sensitivity calculation shown in Fig. 7, we can see that the bars 14, 10, 11 and 12 are the most sensitive, given variations of the tensions, because they would also indicate that these bars are prone, for the injection of reactive power.

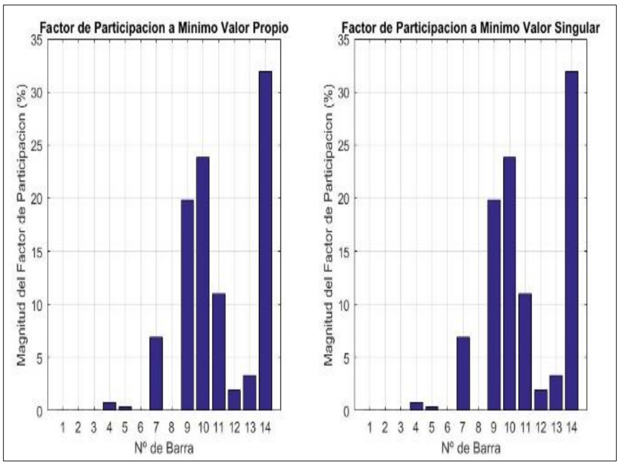

Fig. 6 Participation factors for minimum own value and minimum singular value.

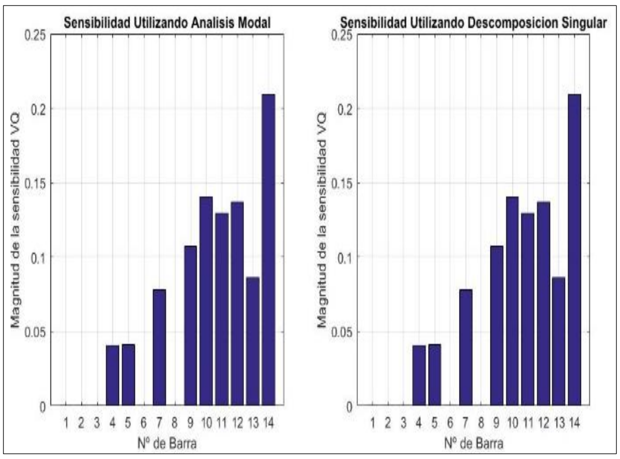

Fig. 7 Sensitivity analysis by modal analysis and singular decomposition methods.

From the methods shown, we can conclude that these methods offer important and similar information, in the detection of critical bars, that reactive power must be injected into the system, so it should be considered a control area, prone to the connection of devices shunt, which will cause the system to be within the region of tension stability.

4. Discussion and Conclusions

These equations correspond to the Linearized representation of the power system around the operating point under analysis.

The expanded Jacobian matrix can be used for local and remote voltage control, which would mean an increase in the number of equations, which will be used, to determine the calculation of Power Flow.

The method proves to be effective, due to the minimum number of iterations that it presents in the convergence process, which would mean that it does not require the performance of multiple tests in order to obtain the desired voltage modules in the bars, because the Expanded Jacobian matrix method, adjust the desired stress values.

The system is stable in voltage, if all the eigenvalues and all the singular values are positive, getting away from the instability of voltage due to an increase by injection of reactive power.

The proposed methodology, together with the techniques of power flow, modal analysis and singular values are a good alternative for the evaluation of the stability of tension, in steady state.

References

[1] B. Gao, G. K. Morison and P. Kundur, Voltage stability evaluation using modal analysis, *IEEE Transactions on Power System* 7 (November 1992) (4) 1529-1541.

[2] A. Sode, Yome N. Mithulananthan, and K. Y. Lee, Effect of realistic load direction in static voltage stability study, in: *IEEE/PES Transmission and Distribution Conference & Exposition: Asia & Pacific*, 2005.

[3] N. Amjady and M. Esmaili, Application of new sensitivity analysis framework for voltage contingency ranking, *IEEE Trans. Power Systems* 20 (2005) (2) 973-983.

[4] V. Ajjarapu and B. Lee, Bibliography on voltage stability, *IEEE Trans. On Power Systems* 13 (February 1998) (1) 115-125.

[5] J. Grainger and W. Stevenson, *Power Systems Analysis* (3rd ed.), México, McGraw Hill, 1996, p. 561.

[6] A. E. Gómez, *Analysis and Operation of Electric Power Systems* (2nd ed.), Madrid, McGraw Hill, 2002, p. 769.