

Mechanics of VAWT

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Abstract: Among much superiority offered by the vertical axis features of a wind mill over the horizontal axis mills, we must consider the fact that the machine is in yaw motion and the wind is not always horizontal with respect to the axis of horizontal machines therefore it has definite advantage. Here we will incorporate a modified Darrieus-type windmill combined with an elevated water storage tank. The VAWT blades thus form a shape of elliptical configuration with connection flanges at the top and bottom of the elevated tank portion. The stem post of the elevated water tank, a cylindrical shell, will also serve as turbine generator support. The windmill will perform various functions, including water pump, electrical generator, air compressor, vacuum pump and refrigeration compressor by means of different gear boxes. Except for the water storage tank, all of the equipment is located inside the bottom cone of the water tank support to provide efficient maintenance and inspection procedures. By adapting the vertical axis wind turbine (VAWT) system, improved wind power efficiency and reduced maintenance costs are expected. Methods of structural design procedures are provided after relevant pressure calculation.

Key words: darrieus-type windmill, VAWT, HAWT, Ellipsoidal-shaped elevated water tank, gear boxes, water pump, air compressor

1. Introduction

The majority of wind mills are of horizontal axis all over the world and the Darrieus-type vertical axis wind turbines are rarely used, but it has been in existence for decades since it was first invented by the French engineer G. J. M. Darrieus in 1920. It has definite advantages in the efficiency of turbine operation as well as aerodynamic performance.

In this investigation we will use a spheroidal-type elevated water storage tank as a basic frame of supporting structure with vertical axis wind turbine (VAWT)-type blades attached to the existing or new tanks.

As shown in Fig. 1, top flange is installed on the top of the spheroidal shell and the bottom flange is installed at the transition from the spheroid to the cylindrical support.

The vertical blades are connected to the flanges at

both ends, thus the flanges are free to rotate horizontally. The connections between the flanges and blades will then take static and dynamic loads due to the blades taking the wind pressure as static load due to the weight of water being stored, the maximum wind loads specified by the codes of American Water Works Association.

The cylindrical shell of the water tank is acting as the single support for all loads, including the weight of water to be stored and the loads due to the wind turbine blades. The round shape of the tank came from the membrane theory of shell structures so that the shell plates will take tensile stresses only.

Installing the vertical blades around the circular shape of storage tank will be beneficial to the nature of axisymmetric configuration.

The blades are all tied by a horizontal stabilizing hoop member and vertically connected at the top and bottom before deflection due to the wind. The maximum deflection is anticipated at the midpoint when the blades take wind pressure and rotate.

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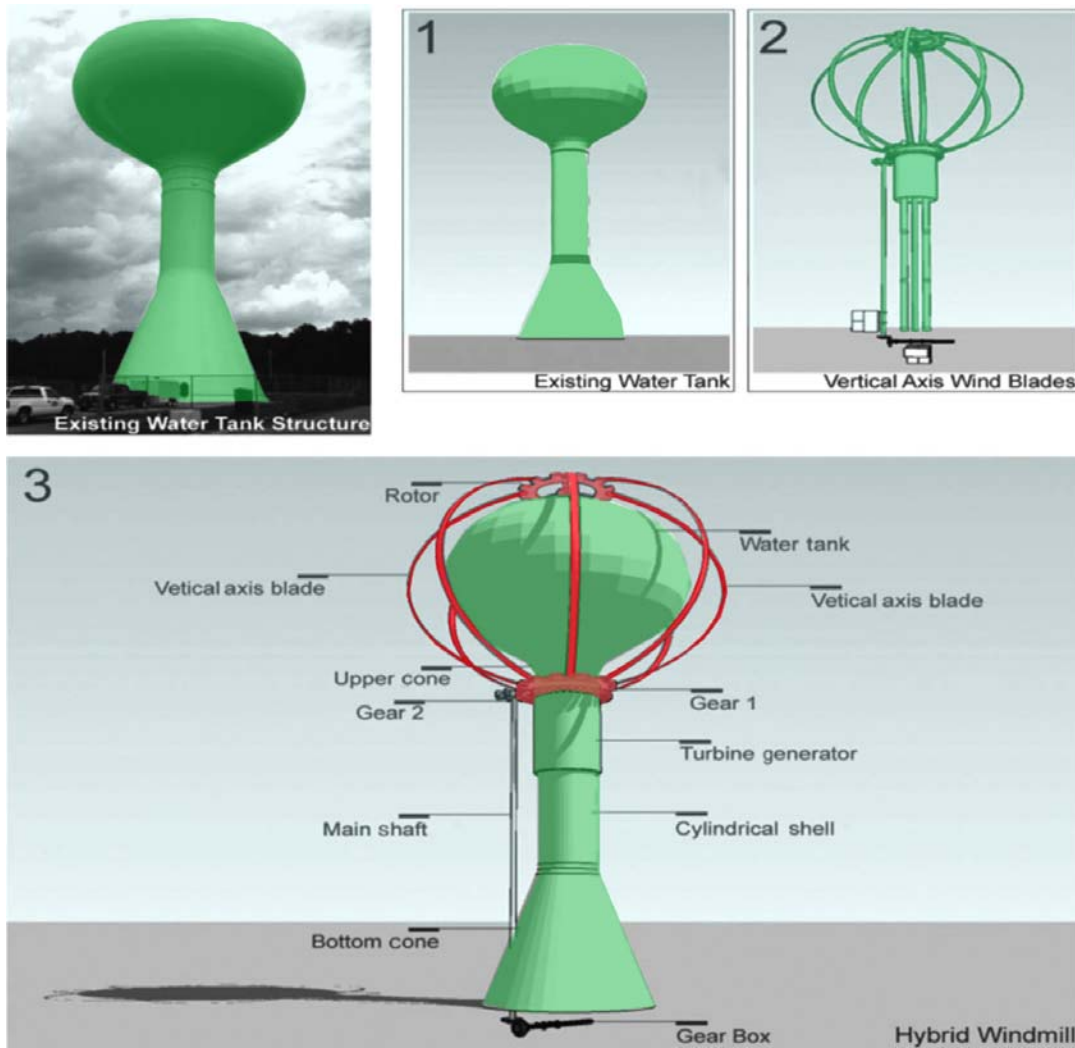


Fig. 1 VAWT combined with existing elevated water tank.

The wind velocity at the higher elevation is estimated is governed as follows:

$$\frac{V}{V_0} = \left(\frac{H}{H_0} \right)^\alpha \tag{1}$$

The notation V and H are the wind velocity and the height respectively. The value α is specified by the Eqs. (2) and (3).

The pressure acting on the blade surface is given by:

$$\frac{P}{P_0} = \left(\frac{V}{V_0} \right)^3 \tag{2}$$

From the Eqs. (1) and (2), we can build the relationship between the pressure and height as follows:

$$\frac{P}{P_0} = \left(\frac{H}{H_0} \right)^{3\alpha} \tag{3}$$

The torque can be geared to various machines, including the turbine generator, air compressor, refrigeration compressor, vacuum pump etc. Because of the vertical arrangement of blades in VAWT, the blades will take wind power from all direction, which is of great advantage.

We will also find greater torque at the bottom flange due to the long arm length provided by the spheroidal shape flange.

With sufficient torque created by the blade system, we can then store the wind energy by means of charging an air compressor and battery together with refrigeration compressors in addition to pumping the water up to the level of the tank.

The system, however, requires intensive studies on the structural integrity of each component. This paper describes the limited basic mechanics and it is noted that further theoretical as well as experimental researches are necessary. Among other components, the importance on the design of the flanges should be emphasized together with the blade connections and the dynamic response.

2. Blade Stress Analysis

The blades are of high strength, lightweight materials, thus advanced composite materials being used in space structures are highly recommended [1, 2].

The composite materials consist of fibers with high tensile strength embedded in resin; they are therefore non-homogeneous anisotropic materials [1].

Anisotropic materials are described in the theory of mechanics by the generalized Hooke's law:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & \dots & E_{16} \\ E_{21} & E_{22} & \dots & E_{26} \\ E_{31} & E_{32} & \dots & E_{36} \\ E_{41} & E_{42} & \dots & E_{46} \\ E_{51} & E_{52} & \dots & E_{56} \\ E_{61} & E_{62} & \dots & E_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad (4)$$

where s and t are normal and shear stresses and (1) and j are normal and shear strains, respectively. E_{ij} are Young's modulus of different directions, where $i = 1, 2, \dots, 6$ and $j = 1, 2, \dots, 6$ and also $E_{ij} = E_{ji}$ from the equilibrium condition.

The vertical blades are as shown in Fig. 4; half circular shape channel laterally bent according to the configuration of spheroidal water tank located inside of the blades, Figs. 1, 5 and 6.

There are eight blades, all connected by 1/2 inch x 12 inch flat bar of horizontal stabilizer as shown in Figs. 4 and 6.

Each blade is subjected to wind pressure as shown in Fig. 2 and must be designed for static and dynamic loads due to the incoming wind pressure and vibration of the blade rotation.

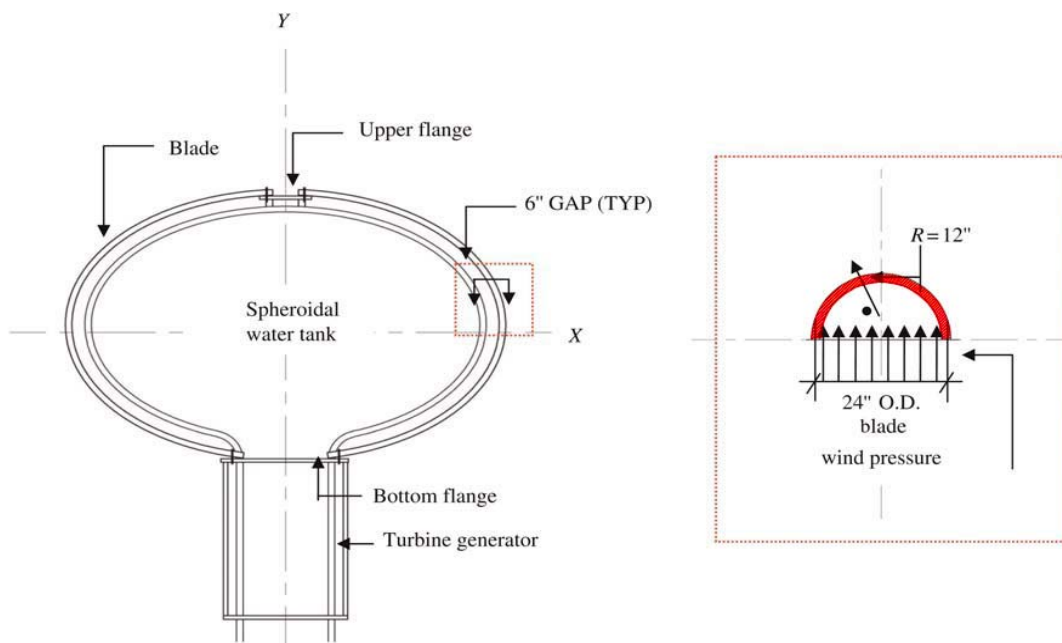


Fig. 2 Wind pressure distribution on the blade section.

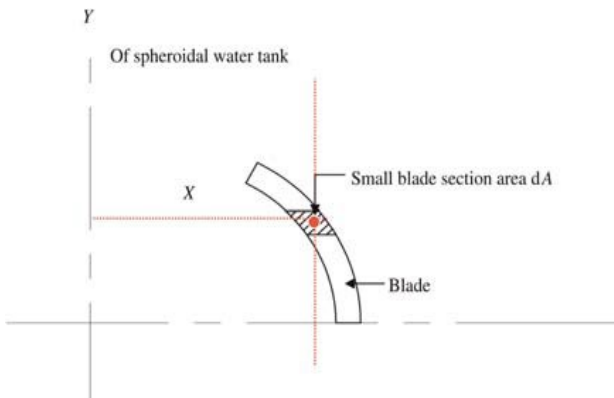


Fig. 3 Torsion due to the pressure on the blade.

The incoming wind pressure acts as a uniform lateral load on the curved beam clamped at both ends of blade

flange connections at the top and bottom as shown in Fig. 4.

The blade or curved beam in this case was then subjected to severe torsion stress in addition to flexural stress, and further investigations are required.

Dynamic analysis is also necessary, not only for the blade stress analysis, but also for the flange design as well as the flange blade connections.

The effect of installing the VAWT system on the existing water tank structures shall be carefully examined and must be reinforced by attaching the reinforcing stiffeners.

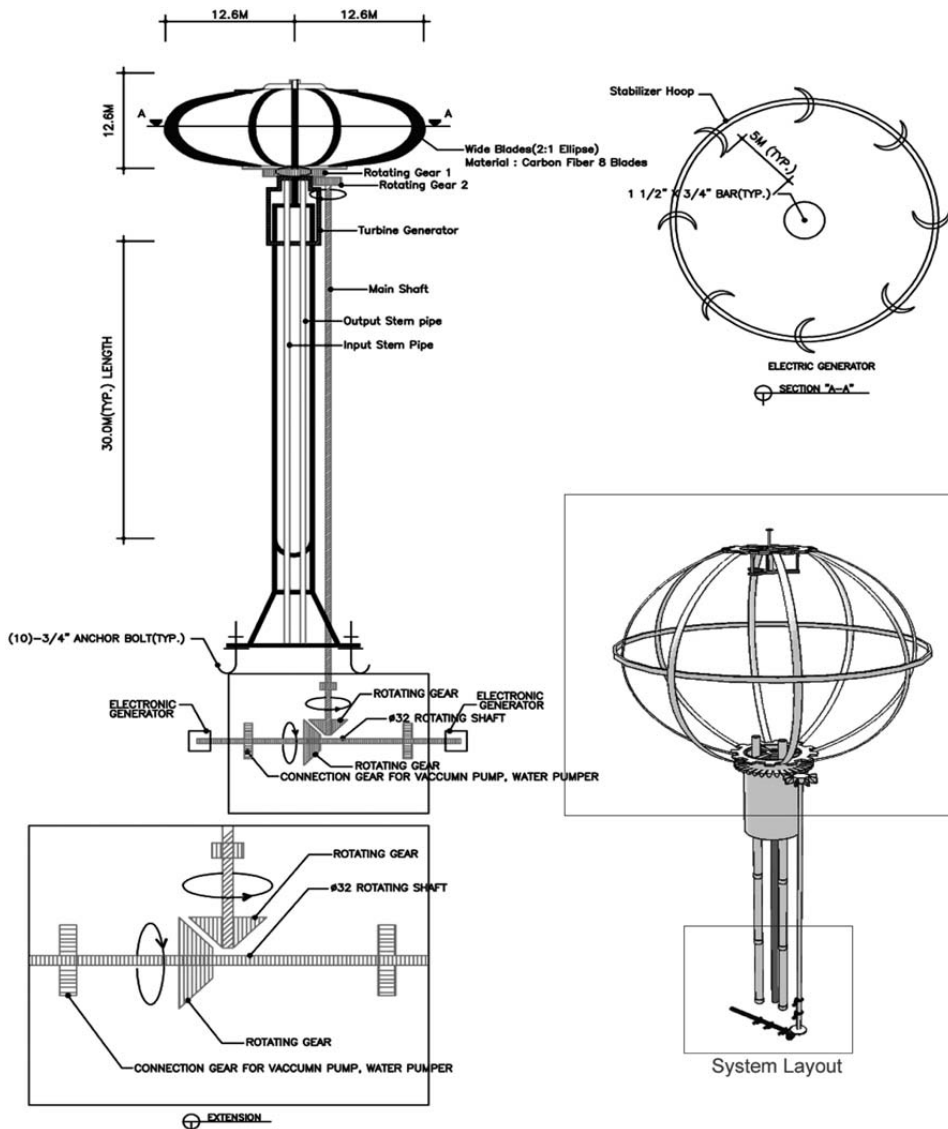


Fig. 4 Hybrid windmill layout.

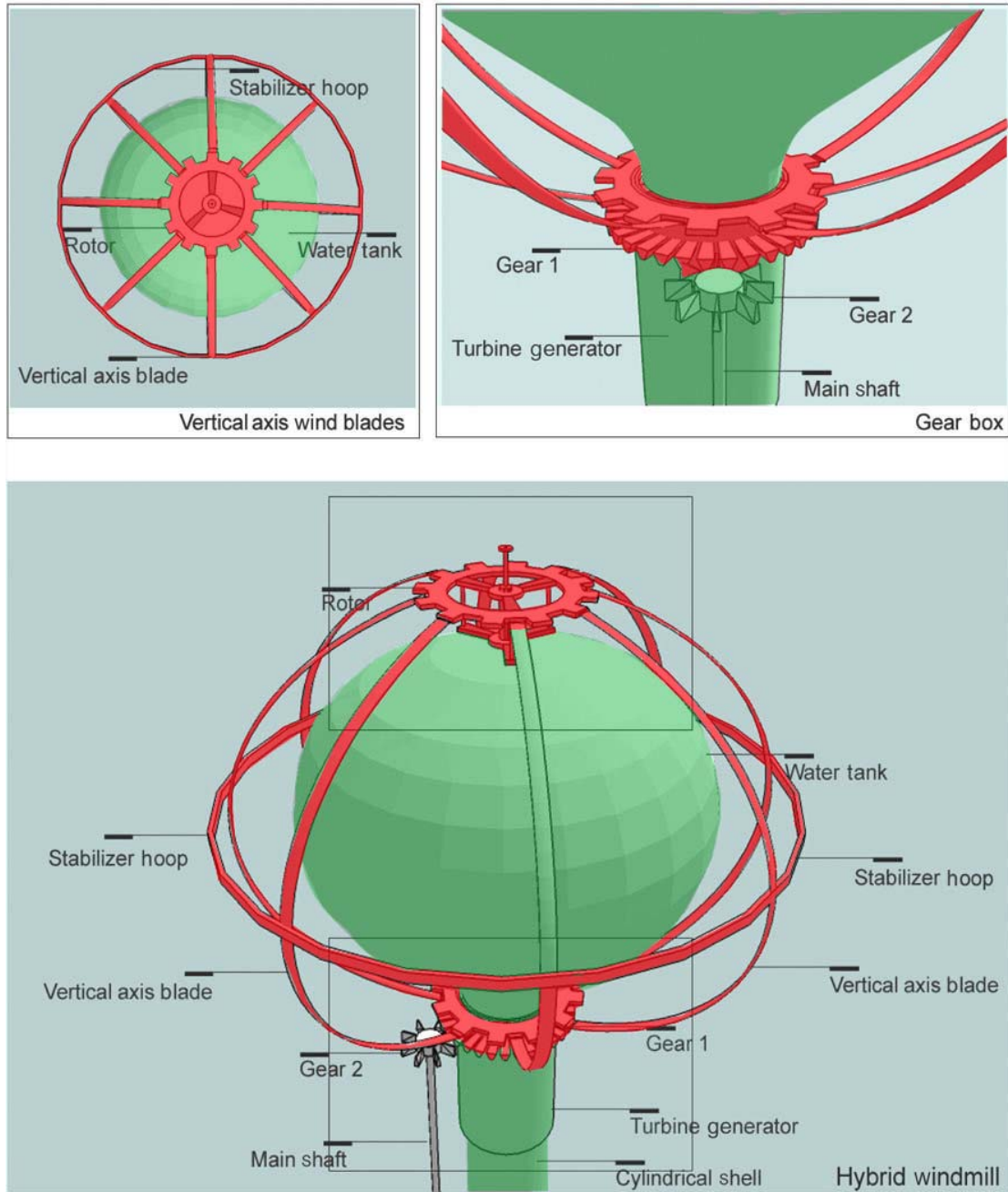


Fig. 5 Details of blade connection.

The loading function $[P]$ is time dependent and the time history of wind can be applied, thus the maximum dynamic stress can be obtained against the wind frequency spectrum.

The torsion forces acting on the flange-blade connection can be computed to aid its design.

The torsion force is obtained by the following integration:

$$T = \int px \, dA \tag{5}$$

where P is unit wind pressure acting on a small blade section of dA . The integration is performed simply by setting the configuration as follows:

$$\begin{cases} x = a \cdot \cos \theta \\ y = b \cdot \sin \theta \end{cases} \tag{6}$$

where a is long radius of the blade b is the short radius respectively.

We can then convert the integration as follows:

$$T = 4 \int_0^a px \cdot \sqrt{1 + y'^2} dx \quad (7)$$

Substituting the Eq. (6) will result

$$T = 4p \int_{\frac{\pi}{2}}^0 a \cdot \cos \theta \sqrt{1 + \frac{b^2}{a^2} \cdot \frac{\cos^2 \theta}{\sin^2 \theta}} \cdot a \cdot (-\sin \theta) d\theta \quad (8)$$

Performing the remaining substitution and integration, we will obtain the governing equation for the torque that we need.

$$T = 4p \cdot a \cdot \sqrt{a^2 - b^2} \cdot \left[\frac{t}{2} \sqrt{t^2 + \frac{b^2}{a^2 - b^2}} + \frac{1}{2} \cdot \frac{b^2}{a^2 - b^2} \cdot \ln \left(t + \sqrt{t^2 + \frac{b^2}{a^2 - b^2}} \right) \right] \Big|_0^1 \quad (9)$$

The results of computation is shown in the Fig. 7.

3. Connection Design

The most favorable connection between the vertical blades and the flanges is a bolted type at slip critical rating.

The connections will take combined shear and tensile stress that can be computed from the basic theory of mechanics.

The reactions obtained from Eqs. (10) and (11) by inputting the boundary conditions will be applied as an eccentric shear force that will be resisted by the shear strength of the bolts being used.

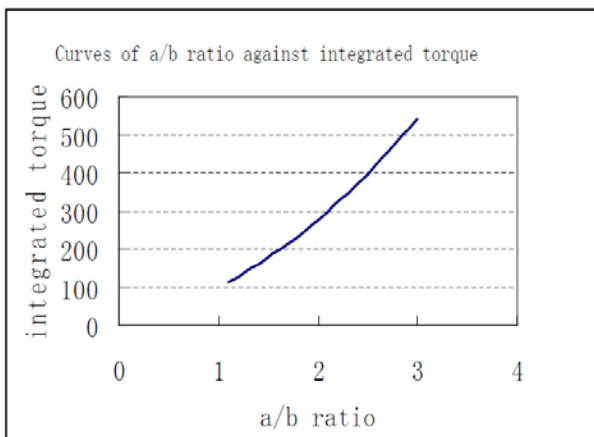


Fig. 7 Total torque.

The total bolt reaction in the X and in the Y directions can then be obtained as follows [3, 4]:

$$R_x = \frac{M_y}{\sum x^2 + \sum y^2}$$

$$R_y = \frac{M_x}{\sum x^2 + \sum y^2} \quad (10)$$

R_v = Circumferential force on blade/number of bolt

where M is the maximum eccentric moment due to the torsion reaction obtained from Eq. (10), R_x is the bolt reaction in the X direction and R_y is of the Y direction and x and y represent the bolt gages and pitches.

The total bolt resisting force is then obtained as follows:

$$R = \sqrt{(R_y + R_v)^2 + R_x^2} \quad (11)$$

R_v = Circumferential force on blade/number of bolt

We are now able to select the material and the size of the bolts by considering the strength of bolts to be used certified by the manufacturers. By using bolted connections, repair and replacement of the blades are allowed. High strength, light weight bolts are recommended [5, 6].

4. Conclusions

Due to the fact that the wind is a time dependent function and its horizontal and vertical directions, velocity and power are continuously changing, the Darrieus-type vertical blades are of higher efficiency compared with the airplane propeller-type horizontal axis wind turbine blades that are mainly designed for horizontal component of the wind.

The wind is of very precious phenomena, can be directly converted to the electric bills which can be equated to cash amount, no reason to waste it. The horizontal axis machines are operated by a component of headwind due to the propellers, which we must give up the other component of wind by twisted angle of blades. As shown in Figs. 4-6, the vertical axis mills will accept 100 percent of headwind energy regardless of the wind direction or off-horizontal

upwind or down wind. It is unfortunate that horizontal machine designers don't realize the truth and consumers simply take it.

We need the electricity and must be generated economically and safely. It is well known that the coal fired or nuclear reactor generated plants are extremely hazardous, no need to repeat it again here. Another important factor of wind turbines are that we don't need fuel or trouble with spent fuel, only generating force is the wind and the wind is unlimited everywhere on earth. No spent fuel issues.

It is time now to recognize the great advantages offered by vertical axis mills.

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