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Proof of the Drawing Possibility Based on the Galois Theory for Teacher Reeducation

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Abstract: This paper aims to develop the high quality teaching materials for teacher remedial education. I did teaching materials development about the drawing figures of the regular pentagon. Specifically, they are Ptolemy's method of constructing an equilateral pentagon for Western mathematics, those of Hirano Yoshifusa and Ajima Naonobu with regard Japanese mathematics and a proof of the drawing figures possibility of the regular pentagon based on the Galois Theory. After lecturing to a teacher with these teaching materials, I was able to really get a favorable reception from expert teachers. This gives an extremely important theoretical viewpoint in developing the high quality teaching materials for teacher remedial education.

Key words: goloistheoy, pentagon, teacher reeducation

1. Introduction

In this paper, I will introduce methods of constructing an equilateral pentagon according to Western and Japanese mathematics respectively, for purposes of teacher reeducation. I will also introduce a proof according to Galois Theory of the figure's constructability. Specifically, I will present Ptolemy's method with regard to Western mathematics and those of Hirano Yosifusa and Ajima Naonobu with regard to Japanese mathematics. I will use cyclotomic fields to discuss a proof of the constructability of an equilateral pentagon. Students often use straightedge and compass to draw figures in school, and not only in math classes. The ancient Greeks were already aware of figures limited to lines drawn with straightedges and circles drawn with compasses.

It is well known that a number of problems regarding non-constructible figures were considered particularly difficult, as it was not known that they were impossible to solve. Three famous examples are the figure of a general angle divided into three equal parts, the figure of pi, and the figure of $\sqrt[3]{2}$. However, constructible figures have also been known of since antiquity. Examples include the figure of an angle bisector, the figure of a vertical bisector of a line segment, and the figure of a square with the same area as a given rectangle. In actual fact, some children are curious to know whether all equilateral polygons are constructible. It is thought that the ancient Greeks believed that the figures of equilateral polygons other than those given above were not constructible. Since Galois Theory had not been formulated in his time, Gauss, in fact, researched the root of the reciprocal equation which the primitive pth root of unity fulfills. In the present, it is possible to prove the constructability of equilateral polygons through Galois Theory. Here I will present a proof of the constructability of an equilateral pentagon.

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2. Evaluation of Japanese Mathematics

It is by no means impossible to address Japanese mathematics in math classes at modern-day schools in Japan. However, it is neither a simple nor an easy issue. Most teachers who have never studied Japanese mathematics are amazed by its quantity and quality. It is logically possible to use it in schools, in every unit from elementary school arithmetic through high school mathematics. Unlike the Meiji era, however, most modern-day schoolteachers in Japan are not practitioners of Japanese mathematics. Therefore, since they know only a small part of Japanese mathematics, it is thought that they inevitably face great difficulty when introducing it into modern school mathematics classes. Even so, all publishers of elementary school arithmetic textbooks include a portion of Japanese mathematics in their texts. It is well known that in the Edo period, when Japanese mathematics was popular, few children disliked math. Japanese mathematics seems overwhelmingly attractive when compared to the dislike many modern-day children have for math and arithmetic. Could one make textbooks based on Japanese mathematics for modern-day children, they would probably be considered effective for school math classes today. I want to introduce, thus, two Japanese mathematicians' method of constructing an equilateral pentagon, a topic taught in school. Construction methods for an equilateral pentagon per Western Mathematics and Japanese mathematics concerning the construction of an equilateral pentagon, I will present Ptolemy's method with regard to Western mathematics and those of Hirano Yosifusa and Ajima Naonobu with regard to Japanese mathematics. I will also introduce a method of proving the constructability of an equilateral pentagon according to Galois Theory. Specifically, it is a proof using a cyclotomic field. This is also significant as a proof of a geometric problem using an algebraic method. As well, the proof of this equilateral pentagon is also meaningful for the interest value of the calculation itself. While demonstrating both the tried-and-true Western method and the Japanese method, which dates to the Edo era, of constructing an equilateral pentagon, I also verified the construction not by geometric but by algebraic methods.

3. Ptolemy's Method of Constructing an Equilateral Pentagon

Ptolemy's method is explained in the Almagest by a quotation from Book XIII Proposition 10 of the "Elements". Below, I will discuss Ptolemy's method of constructing an equilateral pentagon.

As in Figure 1, line segments AB and CD are diameters of circle O.

Line segments AB and CD intersect at right angles at the center point O.

As well, call the midpoint of circle O's radius CD point E. The circle of which line segment EA is the radius intersects with the radius OD of circle O at point F.

At this time, let line segment AF be the length of the side of the equilateral polygon inscribed in circle O.

In fact, in Figure 2, the circle with center A and radius AF intersects with the original circle O at points G and H. Next, let the circle with center point G and radius GA intersect with circle O at point J. Further, let point K be the intersection of circle O with the circle with center H and radius HA. Connecting points A, G, J, K and H, as in Figure 3, will produce an equilateral pentagon.

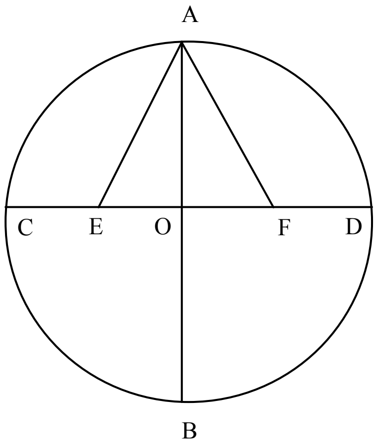

Figure 1

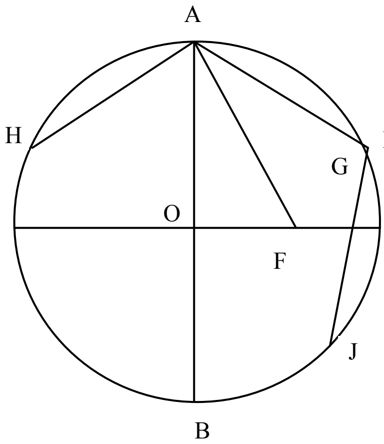

Figure 2

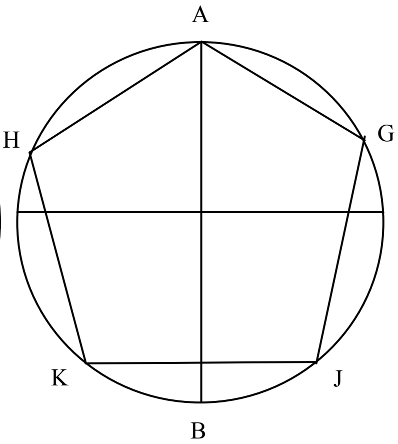

Figure 3

Book IV Proposition 11 of the "Elements" reads "To inscribe an equilateral pentagon in a given circle".

Proposition 10 reads "To construct an isosceles triangle having each of the angles at the base double the remaining one", and is preparation for constructing an equilateral pentagon. Proposition 11 involves drawing the figure in Proposition 10.Its point is that the triangle made from the diagonal and sides of an equilateral pentagon will be an isosceles triangle with an apex angle of 36°, so that if we inscribe a triangle similar to this in a given circle, it will be possible to inscribe an equilateral pentagon on this basis. An equilateral pentagon is determined by its sides and diagonal. Therefore, even without a circle, it is constructible with sides and diagonal. Given side a and diagonal b of an equilateral pentagon, the equation b(b-a) = a2 holds, so if we solve this as a quadratic equation of b, we get b. Therefore, with regard to the given side a, if we construct the bisector segment c's √5 line segment of √5 times c, the length of diagonal emerges, and so it becomes possible to construct an equilateral pentagon. The actual construction of an equilateral pentagon by Japanese mathematicians

4. An Equilateral Pentagon by Japanese Mathematicians

In this section, I will introduce the construction methods of an equilateral pentagon by the two Japanese mathematicians Hirano Yosifusa and Ajima Naonobu.

Hirano Yosifusa's method of constructing an equilateral pentagon is as follows.

As in Figure 4, draw a circle of arbitrary size.

As in Figure 5, draw another circle in congruence with the circle in Figure 4.

At this time, draw the two congruent circles so that they are circumscribed.

As in Figure 6, draw a large circle which circumscribes the two circles in Figure 5.

As in Figure 7, draw a line segment which becomes the diameter of the three circles. Draw the external common tangent of the two congruent circles so that it becomes the diameter of the large circle. Further, connect the center of the two congruent circles, the external common tangent of the two congruent circles, and the intersection with the large circle with line segments.

As in Figure 8, draw another circle which becomes a common circumscribed circle of the two circumscribed congruent circles.

As in Figure 9, draw line segment AB so that it becomes a line segment connecting the two intersections of the common circumscribed circle of the two congruent circles circumscribing the circle in Figure 5.

Call the two intersections A and B, and the line segment AB. Thus, the line segment AB becomes one side of the equilateral pentagon.

In this way, it is possible to construct an equilateral pentagon.

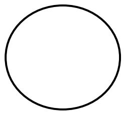

Figure 4

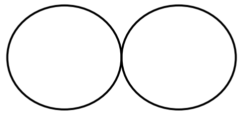

Figure 5

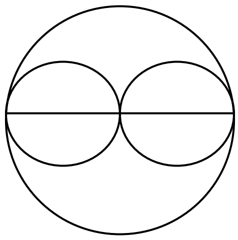

Figure 6

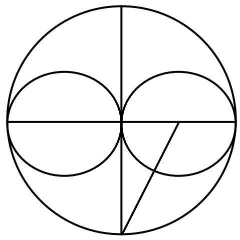

Figure 7

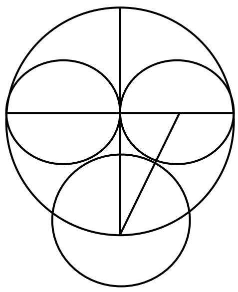

Figure 8

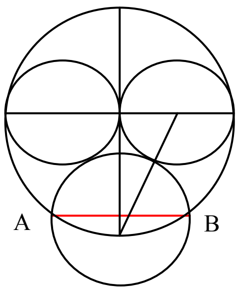

Figure 9

I will hereupon prove that Hirano's method of constructing an equilateral pentagon is accurate.

Proof

It will suffice to show that ∠DBA in Figure 10 has an angle of 18°.

The radius of the largest circle in Figure 10 can be made 1 without loss of generality.

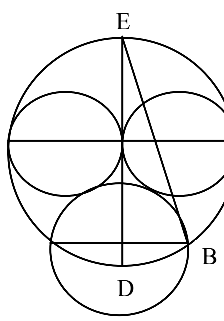

Figure 10

Thus, therefore,

Ajima Naonobu's method of constructing an equilateral pentagon is as shown in Figure 11.

Draw two line segments intersecting at right angles, and call the intersection A.

With intersection A as the center, draw a circle with radius AB, and let point C be the intersection with the line segment drawn to the side in ①.

With point B as the center, draw a circle with radius BA.

Let point D be the intersection of circle B and the vertical line.

With point D as the center, draw a circle with radius DC.

Let point E be the intersection of circle D and the vertical line.

With point B as the center, draw a circle with radius BE.

Let points F and G be the intersections of circle B and the horizontal line.

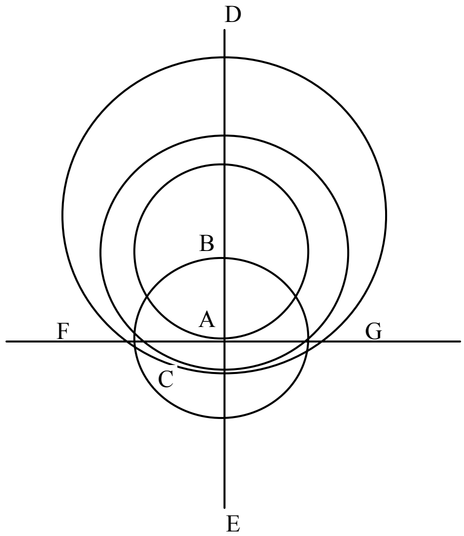

Figure 11

5. Proof of the Drawing Figures Possibility of the Regular Pentagon Based on Galois Theory

I will hereupon prove the constructability of an equilateral pentagon based on Galois Theory.

Proof

As the construction of an equilateral pentagon and the division of the circumference of a circle into five equal parts are equivalent problems, I will prove that the division of the circumference of a circle into five equal parts is possible.

Make ω72° from 5.

As well, let *L* be an extension field.

Here, if we make f(x) the minimal polynomial of ω, it becomes f.

L now becomes the minimum splitting field of f(x).

If we make L the vector space over K, *ω* becomes the base of the vector space.

If we make b an arbitrary member over L, it becomes ζ.

However, ai∈K.

Due to the nature of the nth root of 1, it becomes ω.

The extension order is [L:K] = 4, so the Galois order is 4, and is constructed from the members replacing ω.

If we put σ here, it is expressed as *G* and becomes a cyclic group.

G has a σ2 normal subgroup, the only existent proper normal subgroup.

Trivial subgroups are excluded in this case.

According to Galois' fundamental theorem, an intermediate field of K and L exists.

If we make that M, the intermediate field M becomes the fixed field of group H.

We can find the coefficient conditions thereof as follows.

Thus, it becomes σ when a1 = a4.

As well, when the exponent of ω is larger than 5, it returns to ω with mod. 5.

Therefore, we can write the member α of intermediate field M as follows when we put in a and b.

Put in b.

If we also put in α and β,

Then here, according to the relation between the coefficient and the solution of the quadratic equation, α and β become the solution to the quadratic equation x2.

However, the signs of α and β, as in Figure 12 below, are determined by the connection with ω.

From the above work, M = K(5).

Therefore, when we consider the algebraic equation over M, ω becomes the solution of the equation x2−αx+1 = 0,

and ω2 becomes the solution of the equation x2−βx−1 = 0.

If we can construct α, then when we draw a vertical line from the midpoint of α and the circle center O, we can find the apex ω and ω4, so we can divide the circumference into five equal parts.

And so, the new method based on Galois Theory of constructing an equilateral pentagon is as shown in Figure 12.

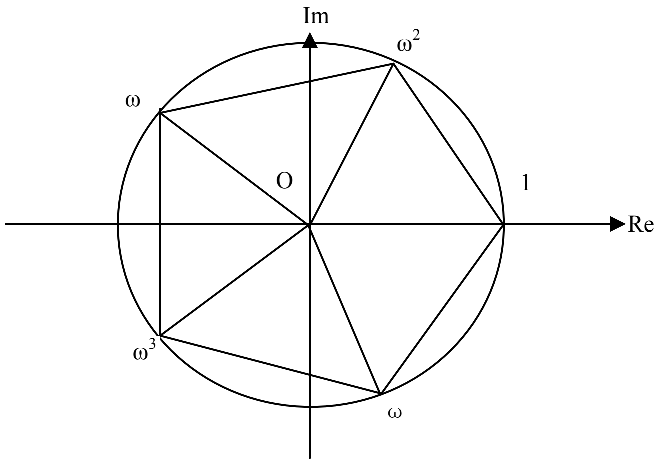

Figure 12

6. Conclusion

In this paper, I have discussed methods of constructing an equilateral pentagon and proofs of its constructability based on modern mathematics. With regard to Western mathematics, I have discussed methods of constructing an equilateral pentagon based on Euclid's Elements and on Ptolemy. I have also discussed the methods of constructing an equilateral pentagon used by two Japanese mathematicians, Hirano Yosifusa and Ajima Naonobu. Because Japanese mathematicians' methods of constructing an equilateral pentagon arose from a unique culture nonexistent in Western mathematics, as far as their legitimacy, proofs were not carried out as they have been in Western mathematics. However, I found their methods to be both original and excellent. Finally, I considered, as a technique in modern mathematics, the constructability of an equilateral pentagon based on Galois Theory, in particular the theory of cyclotomic fields. Although this kind of calculation is often considered to be lacking in interest, I found the proof of constructability of an equilateral pentagon to be an unusual case where the calculation itself was interesting.

At a symposium on the society for mathematics education, Tokyo University of science which was held in Chiba in Japan in August 2011, I spoke on a proof of the drawing figures possibility of the regular pentagon based on the Galois Theory.

Here are the views of two middle and high school teachers who attended this symposium:

(1) Male teacher with 30 years' experience:

"I was able to feel doing mathematics after a long absence. There was not an opportunity blessed with such training at the same time. I thought that such training was important to a school teacher."

(2) Former headmaster with 36 years' experience

"You are excellent. Old generations in their 50s to 60s understood till the last. It might be difficult for the young generation and graduate students. This may depend on the curriculum of college student days."

The mathematics taught in school depends in the end on mathematics proper. For example, when teaching figural symmetry, the teacher cannot get by without the grounding in group theory. As well, the Galois Theory that I used in the proof above is an essential background when teaching equations. For a teacher, at the very least this kind of background in mathematics itself is necessary. In particular, it is thought that teachers must acquire a background in the mathematics proper which relates directly to the materials taught in school. In order to develop high-quality class teaching, it is essential that teacher education provide teachers with as much background in mathematics proper as possible. However, it may not be that the curriculum of a course for preparing students for the teaching profession of a university and the graduate school of current Japan met a high quality teacher training enough. This shows the essential problems of the teacher training. And this gives an extremely important suggestion about the teaching materials in the workshop for teacher remedial education.

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