Modern Environmental Science and Engineering (ISSN 2333-2581)
July 2018, Volume 4, No. 7, pp. 614-628
Doi: 10.15341/mese(2333-2581)/07.04.2018/005
Academic Star Publishing Company, 2018
www.academicstar.us

The Operational Method of Biological Filtration

Béla Tolnai
BioModel Bt. Budapest, Hungary

Abstract: During bank filtration, water is cleaned in a natural way. The well water here is clean, it tastes well and, from a microbiological point of view, it shows an extremely high level of stability. This natural one-stage water filtration process is easy to observe and model. Biological filtration is a complex process depending on a large number of variables. Consequently, for its mathematical description we must rely on dimensional analysis. The model established this way can then be extended to the field of wastewater treatment, based on considerations of similarity. By generalizing the biological mode of action in the above manner, we can lay down the foundations for the theory of biological filtration.

The processes of biological drinking water production and wastewater treatment do not differ from each other. Water purification is based on the same mode of action in both cases, and they are both driven by the same forces. The only difference is that wastewater is considerably more contaminated. Bank filtration has been scaled by nature, whereas wastewater treatment relies on man-made technologies. In no way can it be seen as a mistake if we, after "spying" on bank filtration and finding out about the practices it implements to produce such a high quality of drinking water, should set out to be cleaning our wastewaters in the same way, according to the identified similarity criteria.

Key words: bank filtration, wastewater treatment, similarity, modeling

1. Introduction

László Némedi, the Hungarian microbiologist and former Director of the Microbiology Laboratory of the National Public Health Service in Budapest, once shared the following sentiments with us:

> For over 25 years I have been banging on the doors of our engineers in vain. They just wouldn't recognize the several-fold proven truth: the difference found in the behaviour of **ions** and **microbes** in different media is far more complex than a simple distribution function.

Despite all my respect and my heartiest agreement with this exquisitely formulated insight, my paper aims to set the mathematical foundations for the theory of biological filtration. Throughout the systemization of long-established and more recent approaches to the subject, we will follow a certain axiomatic structure.

Corresponding author: Béla Tolnai, Mechanical Engineer; research areas/interests: biological filtration. E-mail: tolnaibela51@gmail.com.

The axioms of Euclidian geometry form a closed system, from which all geometric theorems can be derived. For the case of biological filtration no such system can be developed. We will, therefore, attempt to pinpoint some basic ideas or, if you like, axioms that cannot be derived formally from any other source, and, based on these, we will formulate some general conclusions related to our field. Bearing in mind that the behaviour of microbes is far too complex to describe by means of a set of functions, we cannot ignore the fact that a simplified mapping of the existing relationships has long been overdue

2. The Process of Biological Water Purification

2.1 Water Quality and Contaminant Load

The biological purification of water is achieved through the decomposition of organic nutrients — also referred to as organic matter or contaminants. Micro-organisms play a decisive role in the

decomposition of molecules, whereby the molecules are turned into water and carbon dioxide. The decomposition of different molecules is performed by different types of bacteria. While water quality is generally determined by the types of molecules the water contains (i.e., what there is to decompose), water contaminant load refers to the concentration of these contaminating molecules (i.e., how much there is to decompose).

Biological drinking water production relies on the same processes as wastewater purification. Although raw and wastewaters to be purified tend not to defer from one another regarding water quality as both contain the same types of molecules, they show a considerable difference in terms of water contaminant load. Contaminant load levels are not directly derived from the concentration of the individual contaminants, but determined as the oxygen demand of the purification process, specified on the basis of certain indicators. This way we can see that the COD and BOD values for wastewater tend to be up to three orders of magnitude higher than the contaminant load level characteristic of natural waters used in drinking water production.

A clear differentiation of water quality and water contaminant load shall thus lead us to our first axiom:

Axiom 1

> Microbes are not aware whether they are participating in drinking water production or in a wastewater treatment process. In other words, the mode of action for biological water purification is always the same, no matter what the purpose.

The experience gathered in the field of drinking water production can, therefore, be relied on while performing wastewater treatment, and vice versa. Consequently, the prevailing clear distinction between the two areas can no longer be justified.

2.2 Bank Filtration: An Easy-to-Observe Example

A well is constructed in the gravel and sand layer on the river bank in order to drain the layer. The depression in the well is generated by means of a pump. The pressure difference required for driving the water through the aquifer into the well is generated by the level difference between the river water level and the well water level (see Fig. 1). This creates a variable velocity flow in the layer, with a flow velocity that increases along the filtration route towards the well.

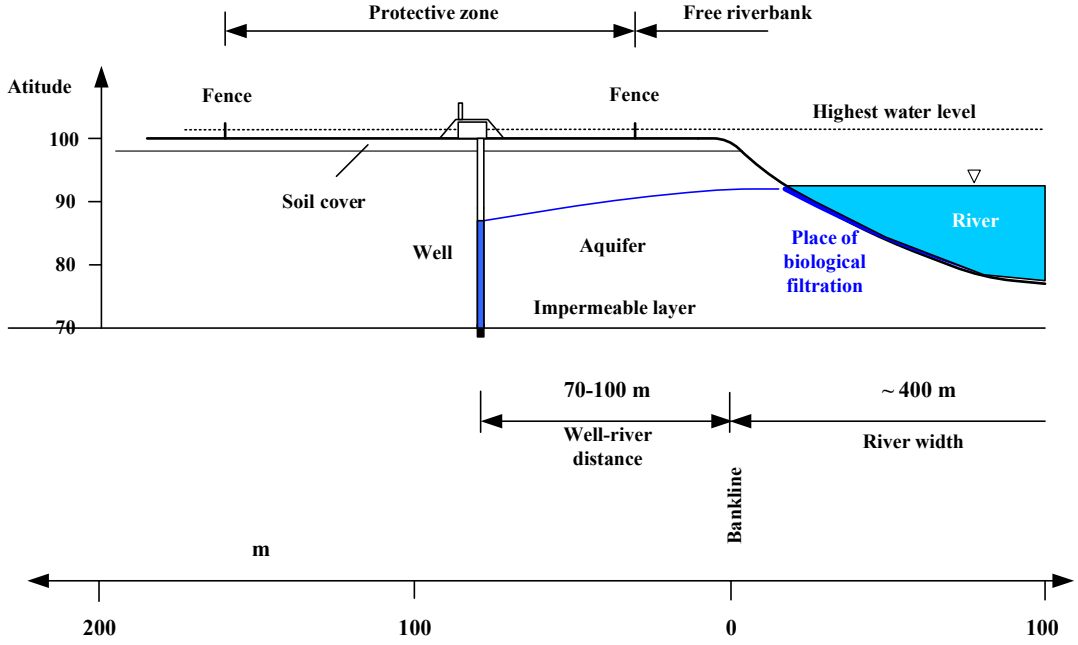


Fig. 1 River - aquifer – well.

At the beginning of the filtration route the velocity of water draining through the layer is rather low, around w = 0.1-0.5 m/day. This low velocity enables the contaminating solved organic matter to diffuse into the biofilm adhering to the surface of the sand grains (see Fig. 2).

The driving force of this diffusion is generated by the concentration difference existing between the concentration levels in the water and inside the biofilm.

Fig. 3 illustrates the separation of contaminants, i.e., the mode of action of water purification. The contaminating molecule is decomposed within the biofilm.

2.3 Generalization of the Observational Study Results

In this section, we aim to extend the observations made in the case of natural biological filtration, and discuss biological filtration in general.

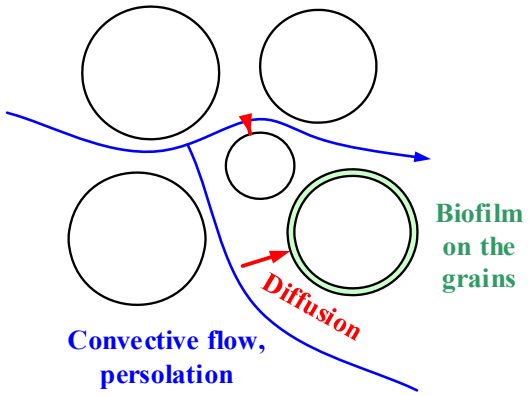


Fig. 2 Convective and conductive flow.

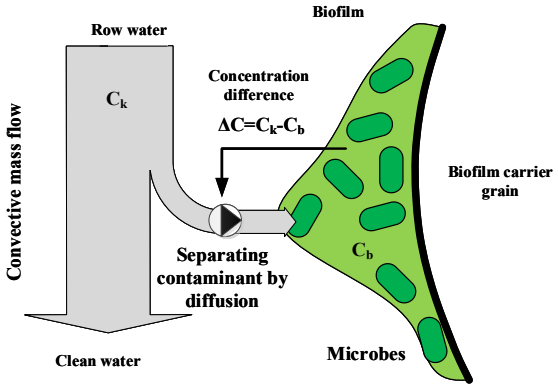


Fig. 3 Separation of contaminants.

The following axiom, frequently described in course books, has aptly been supported by the observations made in connection with bank filtration:

Axiom 2

> Biofilm requires a solid surface to adhere to [10].

In other words, microbes require a carrier surface which they can colonize.

With high water contamination levels, a large number of bacteria is required to perform the purification of the water, which means than an extensive carrier surface must be provided for them to adhere to.

This surface can be immobile, in which case a contiguous biofilm will be formed on it. This is usually observed with bank filtration.

Some carrier surfaces are, on the other hand, mobile, creating a disconnected or fragmented living area for the bacteria. This is usually the case with activated sludge wastewater treatment.

When observed as a sequence, Figs. 1-3 allow us to formulate the following axiom:

Axiom 3

> Biological water purification comprises a two-stage preliminary fluid mechanical process (mass transfer) and of a biochemical process performed within the biofilm. These sub-processes must follow each other in a particular order.

The complete process of nutrient degradation can be divided into three parts, as illustrated in the table in Fig. 4. First, contaminants are transferred to the biofilm by means of a convective flow or persolation. The movement of water in the layer is caused by pressure difference and, with bank filtration, it is maintained by means of a pump or, with other filtration methods, by means of agitation and air injection. A conductive flow or diffusion will then enable contaminants to penetrate the biofilm, through which they will also be separated from the main water stream. The diffusion is driven by the existing concentration difference.

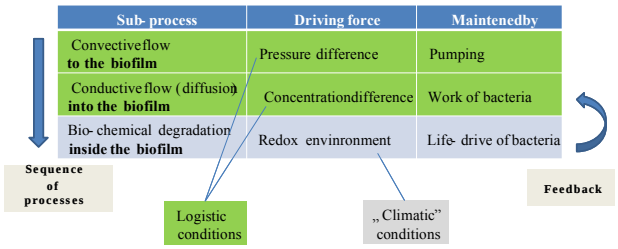


Fig. 4 The bank filtration process.

The separation of contaminants from the water flow actually constitutes the completion of the purification process. These logistic stages relying on the laws of fluid mechanics serve as preconditions (or preliminary stages) to the processes taking place inside the biofilm.

The degradation of nutrients takes place inside the biofilm, and relies on the active 'contribution' of bacteria.

Fig. 4 specifies the driving force as well as the mode of maintenance for each individual sub-process.

During nutrient degradation, the molecules penetrating the biofilm are transformed, and their concentration inside the biofilm is reduced to zero. The difference of concentration levels inside and outside the biofilm is thus continuously regenerated.

The diffusive movement takes place in all directions in the surrounding space, as observed in the case of Brownian motion. The migration of ions is also subject to the laws of diffusion. However, with the help of an electric field, charged particles can be steered into a chosen direction. In order to differentiate this type of guided particle movement from spontaneous diffusion, the former is generally referred to as drift. In our specific case, a one-direction diffusive movement can be observed from the substrate to the biofilm. The driving force is secured through the concentration difference continuously regenerated by bacterial activity, which further causes the diffusion to take place into one single direction here as well.

In Fig. 4, the term life drive is used to describe the drive forcing bacteria to take an active part in the degradation process. In a system-technological sense, the act of degradation provides the required feedback by continuously regenerating the concentration difference. The phenomenon of life drive shall be described later in more detail, following a short detour to some further related issues.

2.4 The Modelling of Bank Filtration

Dimensional analysis allows for the modelling of complex processes and phenomena depending on numerous variables — like the process of drinking water production via bank filtration. It is a generally accepted fact that dimensional analysis tends only to prove useful if

- the mathematical model of the system is unknown, and
- the properties of the processes taking place in the system are known.

For a mathematical modelling of the above process we will rely on the matrix algebra method developed by Tamás Szirtes [4]. As a first step, the application of this method requires the listing of variables characteristic of the process in questions. The variables to be considered here are, accordingly, as follows:

Table 1 Relevant variables.

Variable name	Symbol	SI dimension
Nutrient degradation rate	ΔS	kg/m^3
Substrate concentration	S	kg/m^3
Diffusion coefficient of substrate	D_S	m^2/s
Filtration velocity	w	m/s
Representative particle diameter	d_m	m
Biologically active layer thickness	L	m
Redox potential	E_h	V
Faraday constant	F	As/mol
Absolute temperature	T	K
Molar gas constant	R	$m^2kg/s^2/K/mol$

As a next step, the number of variables must be reduced.

With the model applied for bank filtration [1], as an intermediary result six dimensionless variables can be derived, where the Péclet number (*Pe)* is the physical and L/d_m the geometrical parameter, and the

Ne-coefficient is the similarity criterion of the biochemical process component. The following table summarises the variables that can be influenced on the planner's and operator's side (Table 2).

The formula describing the relevant process can be derived by exponentiating and multiplying together the resulting dimensionless numbers. The application of the Heuristic Method results in Eq. (1):

$$\Delta S = \mu \frac{1}{Pe} \underline{S \frac{L}{d_m} Ne^{1/3}} = \mu \frac{1}{Pe} \varphi \qquad (1)$$

The formula — just as the whole process — can basically be divided into two parts. The part dependent on the Pe-number describes the physical, logistic preconditions of the process, whereas the other factors — contracted into φ — describe the biochemical interactions or, in other words, the *climatic aspects* of the observed phenomenon.

Upon representing the above Eq. (1) in a graph, a set of hyperbolic curves will be obtained (Fig. 5). The individual hyperboles are differentiated from each other by the φ parameter, which comprises various factors.

The interpretation of the μ coefficient is also based on the graph drawn for Eq. (1).

If, from a mathematical point of view, the μ filtration coefficient were a constant, the output values of the hyperbolas — given a small *Pe*-number —would tend to infinity. However, an infinitely high nutrient degradation rate is not imaginable in this range. The output value is expected to be low in the origin, implying that the hypothesis regarding the μ filtration coefficient must be $\mu = \mu(Pe)$. The foldback curve marked in red in Fig. 5 depicts this condition. Nevertheless, in order to determine the dependency of the filtration coefficient on the *Pe* number, we need to

Table 2 Resulting dimensionless variables.

Dimensionless variable	Name
$\Pi_1 = \Delta S / S$	Concentration ratio
$\Pi_2 = w\, d_m / D_S$	Pe, Péclet-number
$\Pi_3 = L / d_m$	Geometric ratio
$\Pi_4 = E_h\, F / RT$	Ne, Nernst-coefficient

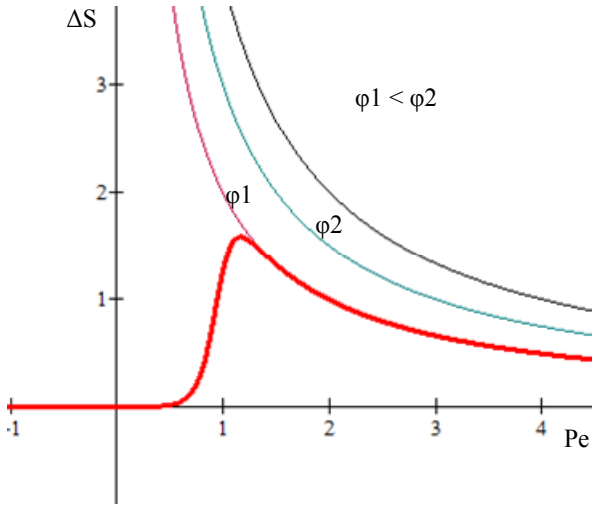


Fig. 5 Nutrient degradation curves.

take a closer look at the nutrient degradation process. The following section, therefore, aims to give a brief overview of how this process works, incorporating the presented information into the theory put forward alongside the above model.

2.5 Logistic Preconditions to Biological Filtration

2.5.1 Interpretation of the *Pe*-number

The logistical conditions of biological filtration are specified by the *Pe*-number as follows:

$$Pe = \frac{w\, d_m}{D_S}$$

where

w [m/s] is the filtration velocity,

d_m [m] is the representative particle diameter (in the case of sand filter, it is equal to the typical particle diameter),

D_s [m^2/s] is the diffusion coefficient of the substrate.

The *Pe*-number is a dimensionless number, comprising three different properties: the filtration velocity (w), which represents the key parameter from an operational point of view; the quality of water to be purified described through its contaminant diffusion coefficient (D_s); and the representative particle diameter (d_m), which specifies the surface area of the carrier.

The *Pe*-number was originally interpreted as the quotient of the convective and conductive velocity:

$$Pe = \frac{w}{\frac{D_S}{d_m}} = \frac{convective\ velocity}{conductive\ velocity}$$

This approach also allows for the determination of preconditions required for an effective degradation, namely that the nutrients transported to the biofilm must be enabled to get inside of it, i.e., the value of *Pe* should be ~ 1.

However, there exists another approach providing us with a more graphic interpretation. According to this, the following formula is derived after the algebraic transformation of the quotient has been performed:

$$Pe = \frac{\frac{d_m^2}{D_s}}{\frac{d_m}{w}} = \frac{\tau}{t} = \frac{diffusion\ time}{retention\ time} \quad \left(= \frac{d_m^2}{D_s}\frac{w}{d_m} = \frac{w\,d_m}{D_s} \right)$$

The formula here is given as a quotient of the time required by the nutrient to cover the dm diffusion distance and the time spent on the outside of the dm -size biofilm-carrier particle.

In order to facilitate an effective nutrient degradation, these two times must be approximately equal (Pe ~ 1). If Pe < 1, there shall not be enough nutrients reaching the biofilter, whereas if Pe >> 1, the nutrients are bound to pass by the biofilm instead of penetrating it.

2.5.2 Calculating the Pe-number

Calculating the *Pe*-number may sound like a simple task to perform. However, determining and interpreting the individual factors typical of the different water filtration methods can raise various problems.

Representative particle diameter

In the case of materials with an inner surface structure in addition to the outer surface, various geometrical aspects must be considered before determining the equivalent (d_e) and representative (d_m) particle diameter [2]. This specific group of materials includes, for example, activated carbon and zeolite.

The individual biofilm carriers can be ranked according to their equivalent and representative particle diameters (Fig. 6).

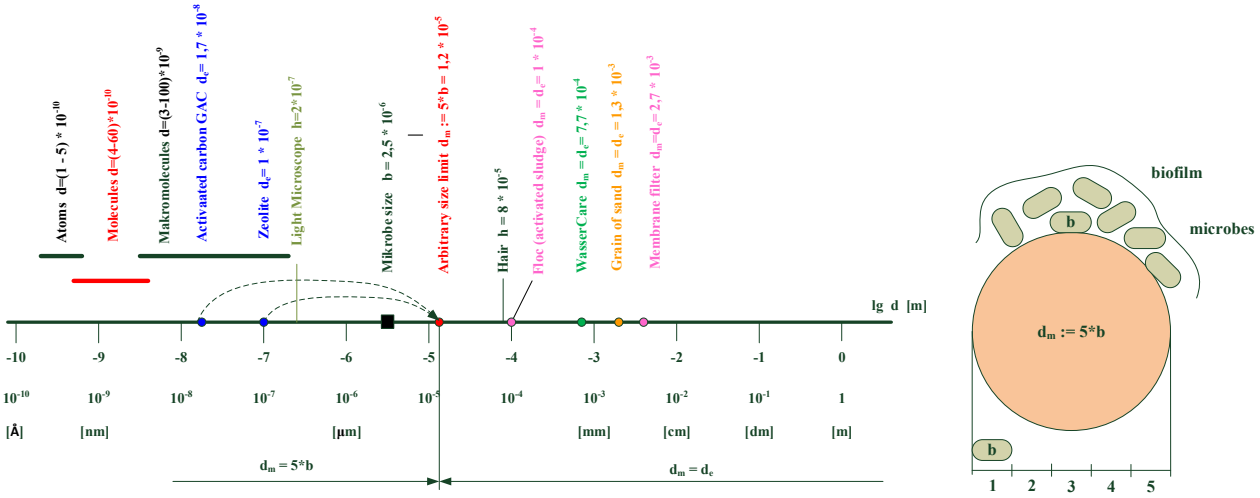


Fig. 6 Equivalent and representative particle diameters.

Axiom 4

> The equivalent and representative particle diameters do not represent actual particle sizes. They merely describe the surface available to the biofilm to colonize.

The smaller the equivalent particle diameter, the larger the available surface. Due to the size of bacteria, the actual inhabitable surface tends to be less extensive and is, therefore, described by the representative particle diameter (d_m).

The applied logarithmic scale indicates that there are great distances between the sizes representing the individual filtering media. We can easily observe that the characteristics plastic biofilm carrier products currently available on the market tend to show (e.g.,

WasserCare and Danpak) are still far from meeting the arising demands, given the fact that their specific surface area is rather small.

Consequently, for the purposes of biofilm carrier medium the most ideal choice are materials the representative particle diameters of which slightly exceed the size of the microbes.

Filtration velocity

In the case of bank filtration, the filtration velocity is easily determined or estimated as the biofilm carrier filtering medium is immobile whilst the substrate carrying the nutrients is moving.

However, it is not impossible for both the surface populated by the bacteria and the nutrient carrying substrate to be moving at the same time. This is usually the case with activate sludge water treatment systems. Regarding diffusion, the relative velocity of the two moving media must be taken into account, which property is not easy to define. An additional challenge is represented by the fact that relative velocity is constantly changes inside the reactors.

Diffusion coefficient

The value of the diffusion coefficient is dependent on the size of molecules to be degraded. The difference between the actual lowest and highest values usually falls within the same order of magnitude. Although these values can vary in the case of the different individual substrates, this difference tends not to be significant with any of them. Consequently, the denominator of the *Pe*-number — as opposed to its numerator — is easily estimated.

The diffusion coefficient, however, is dependent on thermal conditions: if temperatures drop, its value will be reduced. The thermal coefficient remains the same for every type of material.

2.6 Biological Properties of the Filtration Process

In Fig. 4 the life drive of the bacteria is given as the driving force of the biochemical process. This assumption, however, is based on deep root causes, which can be understood with the help of the Michaelis-Menten and Monod kinetics.

2.6.1 Cell Metabolism

The nutrient degradation process taking place inside the biofilm seems simple, should we only be interested in the fact whether or not it has been completed. Nevertheless, if we examine the process in more detail, it will appear to be a lot more complex. In order to facilitate a better understanding of the relevant system technological correlations, the mode of action of the process — so well described by biologists — should be overviewed here briefly.

Cells need energy to maintain their life functions, as do all other living organisms. The degradation of nutrients represents an exothermic process, and the drive to "acquire" the released energy is shown in the life drive.

In order for a molecule to be degraded, it needs to be activated first. The required $\Delta E_{activation}$ activation energy is absorbed from the environment. In the presence of a catalyst, the required activation energy is considerably lower (see Fig. 7). In biological systems enzymes can act as catalysts.

Within the relevant reaction time the substrate is turned into decomposition products, which is an energy releasing process. Getting access to this released $\boldsymbol{\Delta E}$ energy represents the actual drive of the nutrient degradation process.

The process of cell metabolism is described through the Michaelis-Menten enzyme kinetics. This model, developed at the beginning of the 19th century, has identified the role of enzymes in the process, and it

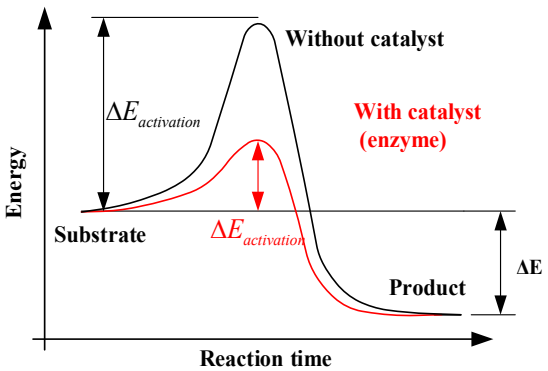


Fig. 7 Reducing the activation energy.

illustrates the process of molecule degradation by means of geometric structures.

Any given type of enzyme is only capable of decomposing one given type of substrate, as indicated in Fig. 8, through the geometric shapes that fit into each other like "key into the lock". This leads us to our following axiom:

Axiom 5

> In order for the degradation process to take place with a high probability, it must involve enzymes "matching" the substrate.

The reversible and irreversible formation phases of the substrate-enzyme-complex-product stages can be described through an equation and differential equation system.

There is a solution to the algebraic and differential equation system represented in Fig. 9. As a result, the reaction rate, the formation rate of the product, will be derived, which is a dependant of the substrate content (see Fig. 10).

The enzyme kinetic model is simple, and it provides an excellent phenomenological description of the process. The v_{max} and K_m parameters can easily be specified. The relevant saturation properties and the

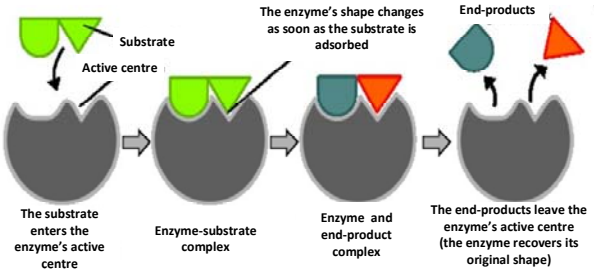


Fig. 8 Substrate-enzyme-complex – product.

$$E + S \underset{k_{-1}}{\overset{k_1}{\leftrightarrow}} ES \xrightarrow{k_2} P + E$$

E	enzyme
S	substrate
ES	complex
P	product

$$\frac{d[ES]}{dt} = k_1[E][S] - (k_{-1} + k_2)[ES] = 0$$

$$[E] + [ES] = [E]_0$$

$$[ES] = \frac{[E]_0[S]}{[S] + \left(\frac{k_{-1} + k_2}{k_1}\right)}, \qquad \frac{k_{-1} + k_2}{k_1} = K_m$$

$$\frac{d[P]}{dt} = k_2[ES] = \frac{k_2[E]_0[S]}{[S] + K_m} = \frac{v_{max}[S]}{[S] + K_m}$$

Fig. 9 The differential equation system of enzyme kinetics.

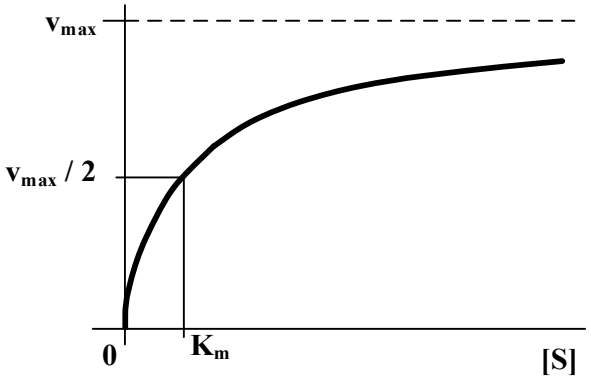


Fig. 10 The reaction rate as the result.

reaction rate growth are described through the K_m half-saturation constant.

As we might remember from our biological studies, degradation is not a one-stage process. Multi-stage processes like this can be observed with nitrification and denitrification, where various different phases follow each other. Here the product produced during the first reaction will act as the substrate in the next phase. However, with multi-stage degradation, the danger is that the process might be stopped before it is completed, which is bound to result in unwanted or even toxic matter being left behind in the water. This might have led to the assumption according to which the process of biological water filtration is highly difficult to control.

The degradation process is often described in the form of stoichiometric equations as well. In an oxic environment the end-products are mostly water and carbon dioxide, which are produced through the oxidation of hydrogen and carbon. Although in principle oxidation works very similarly to a combustion process, the energy released during the reaction is not heat energy — as typical with combustions producing flame, but a form of chemical energy that provides the cell with the energy it requires to maintain its life functions.

Based on the above, the following conclusion can be drawn:

Mechanical filters retain contaminants in the substrate, whereas biological filters "burn them up" on

the spot. Therefore, whilst mechanical filters — including membranes — require regular cleaning, biological filters are mostly self-cleaning. These facts definitely speak for a wider application of biological filtration, or even make it appear preferable to any other method.

2.6.2 Reproduction of Microbes

Bacteria are one-celled organisms, with components like proteins, nucleic acids, lipids and water. Most of its protein content consists of enzymatic proteins, which — as described in the previous section — act as catalysts in the nutrient degradation process and, consequently, are essential for the reduction of the required activation energy.

Bacteria — just as any other living organisms — are capable of reproducing themselves. Bacterial reproduction tendentially takes the form of division. The model describing the kinetics of microbial reproduction was based on the Michaelis-Menten enzyme kinetics system, and was created by Monod in 1942, around half a century after their original theory has been published.

According to the Monod model, binary fission can be described by means of an exponential function, the steps to which are illustrated in Fig. 11 below.

Following the n^{th} generation, a microbe cell number of

$$x = x_0\, 2^n$$

will be obtained. After rearranging and expanding the equation, the applicable formula is as follows:

$$\ln x = \ln x_0 + n \ln 2 \quad \rightarrow \quad n = \frac{\ln x - \ln x_0}{\ln 2}$$

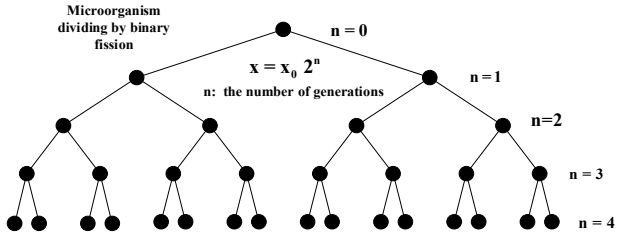


Fig. 11 Binary fission.

The number of generations can also be described as a quotient of duration and generation time:

$$n = \frac{t}{t_g}$$

Generation time is the doubling time of a cell, and it gives information on the reproduction rate of the given cell. The shorter the generation time, the higher the degradation rate.

Although this approach is graphically elaborate, it does present us with various difficulties when attempting to establish the difference of cells numbers in consecutive generations. That is why the process is tendentially described by means of a differential equation instead, as put forward by Monod:

$$\frac{dx}{dt} = \mu_M x \qquad (2)$$

The constant relative growth rate of reproduction is represented by the μ_M coefficient[1], which shows how fast cells multiply from one generation to the other. The solution of the differential equation leads us to an exponential function. The curve for this function approaches infinity with increasing durations of time. Consequently, it is only the beginning of the curve as well as the beginning of the exponential growth phase that is generally used to describe the relevant growth rate.

The exponent of μ_M can be determined though measuring. Its value tends to bear a saturation property depending on the substrate content.

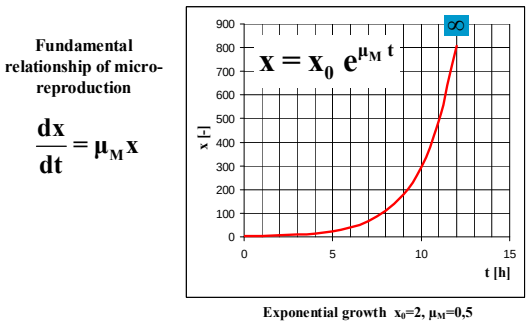


Fig. 12 Reproduction relative growth.

[1] μ_M is not to be confused with the μ filtration coefficient!

There is a formal similarity between the Michaelis-Menten reaction rate and the substrate dependent exponent of Monod kinetics. The growth rates of the curves can be described with the help of the K_m and K_s half-saturation constants (see and compare development properties for Figs 10 and 13).

Microbial growth rate can be measured in hours. Although the $\mu_{M,max}$ saturation level goes up with the increase of dissolved oxygen in the water, the growth rate depending on substrate content — or the slope of the saturation curve — basically remain the same, as shown in Fig. 14.

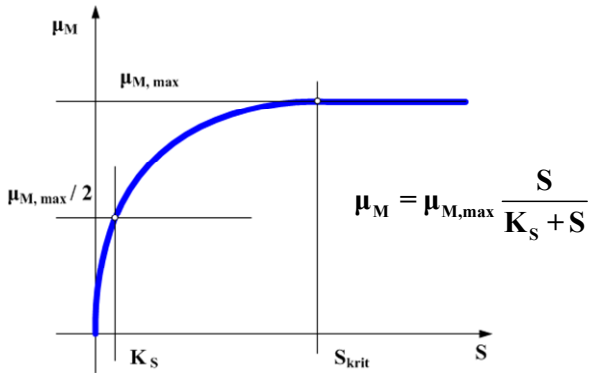


Fig. 13 The Monod-factor.

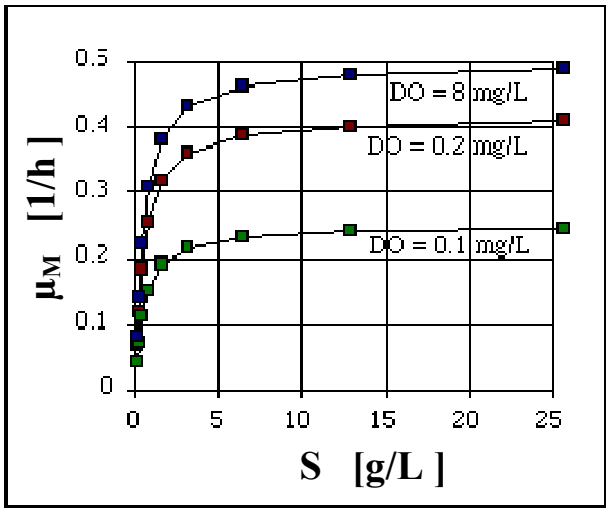


Fig. 14 Monod-factor as a function of dissolved oxygen [9].

The slope of the saturation curve is determined by the type of substrate as well as by the type of reproducing bacteria.

The relevant generation time is easily established through the equation of the following general solutions:

$$x = x_0\, 2^n \quad \text{and} \quad x = x_0\, e^{\mu_M t},$$

which means that there is evidence for a functional relationship between the Monod-factor and generation time:

$$t_g = \frac{\ln 2}{\mu_M} = \frac{0{,}693}{\mu_M}$$

However, with closed systems there is a limit to growth, which, at this point, requires us to set aside the Monod-kinetic approach based on constant growth. This will lead us to an improved version of differential Eq. (2),

which is exponentially "slowed down" in its growth:

$$\frac{dx}{dt} = \mu_M x\,(1 - \frac{x}{K}) = \mu_M x - \mu_M \frac{x^2}{K} \qquad (3)$$

This differential equation has an analytical solution as well [5], which is known to mathematicians as the Logistic Function:

$$x = \frac{K}{1 + x_0 e^{-\mu_M t}} \qquad (4)$$

The curve of the Logistic Function (see Fig. 15) does not approach infinity, and it incorporates an exponential growth phase. Another important characteristic of this function is to be seen in the fact that its output in the origin is low, but not equal to zero.

Biofilms capable of adhering to solid surfaces go through several phases in time (see Fig. 16). The numbered consecutive phases shown in the schematic representation below, i.e., attachment and adhesion (1-2), maturation (3-4) and dispersion (5), give evidence of the dynamic nature of the biofilm's existence. The reproduction and death phases can also be identified in the microscope images.

Fig. 17 shows the individual phases of reproduction and death. The accelerating and decelerating growth

phases are followed by a stationary phase and then by the death phase. The death phase is linked to biofilm dispersion.

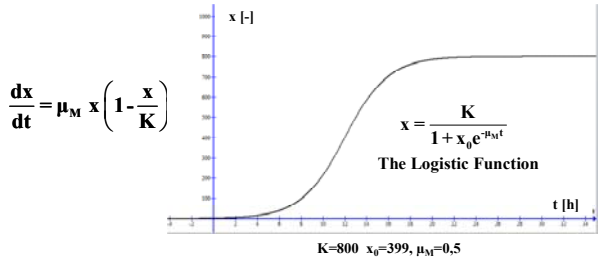


Fig. 15 Logistic function.

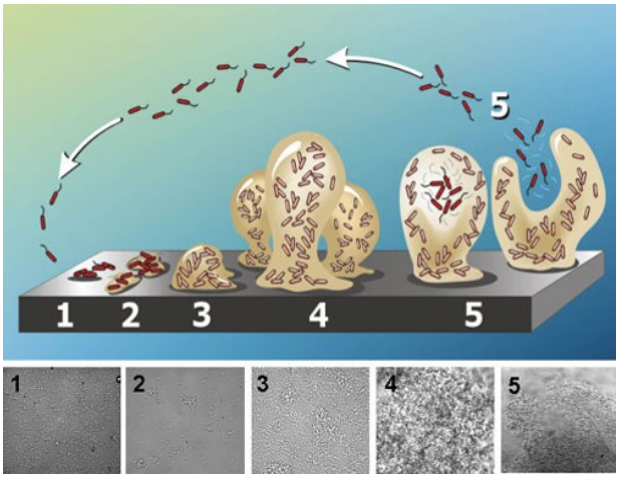


Fig. 16 Life cycle of the biofilm [10].

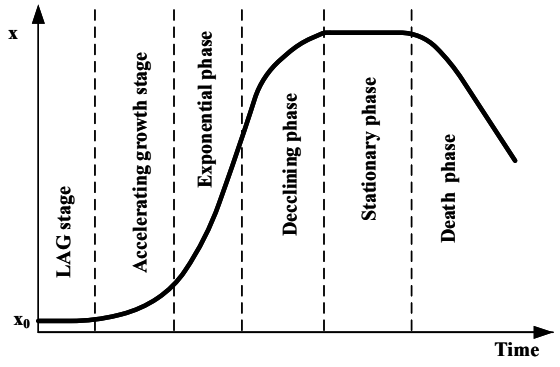


Fig. 17 Reproduction and death.

The above reproduction phases — except for the death phase — can be paraphrased in the language of mathematics with the help of the Logistic Function. In order to perform this "translation", we merely need to select and measure the x_0, K and μ_M parameters correctly. The death phase, however, cannot be described by means of the Logistic Function, just as the section of the Monod-kinetic curve approaching infinity was found unsuitable for the same purpose.

2.7 The Theory of Biological Filtration

Let us now return to our model developed with the help of dimensional analysis, which was comprised in the mathematical formula Eq (1). As we have mentioned before, in this formula the μ filtration coefficient cannot be constant, as with small *Pe*-numbers the curve tends to infinity. Nevertheless, with the assumption of the $\mu = \mu(Pe)$ functional relationship, the hyperbolas can be "turned back". A modification to the curve shape is not only desirable from a mathematical point of view — the considerations put forward along with the interpretation of the *Pe*-number also seem to support its necessity.

First, let us **formally** identify the filtration coefficient by applying the Logistic Function of microbial reproduction:

$$\mu = \mu(Pe) := \frac{\beta}{1 + a\, e^{-b\,Pe}} \tag{5}$$

where

β is the proportionate fraction of the filtration coefficient, and its value can be established by means of measurement, and

the a and b parameters can be derived from the conditions specified above for the *Pe*-number, i.e. that the function around $Pe = 1$ should show a maximum value, whilst in the origin the output value should lie slightly above zero. In order to fulfil these conditions, the approximate values for a and b must be $a = 100.000$ and $b = 12$.

β , a and b are all dimensionless parameters.

Substituting the Eq (5) into the Eq. (1), the functional relationship for nutrient degradation will be obtained.

$$\Delta S = \frac{\beta}{1 + a\, e^{-b\,Pe}} \frac{1}{Pe} S \frac{L}{d_m} Ne^{1/3} = \frac{\beta}{1 + a\, e^{-b\,Pe}} \frac{1}{Pe}\, \varphi \tag{6}$$

Through the formal synchronization of the Logistic Formula and the filtration coefficient, the infinite nature of the hyperboles will "disappear", and the nutrient degradation function will show a maximum output value at around $Pe = 1$.

The filtration coefficient is a characteristic of the nutrient degradation model. Through the Logistic Function we can describe microbial reproduction in a closed area. By means of a formal synchronization of the two notions, we have, in fact, joined the considerations arising from the two different approaches. The results of this can be illustrated and interpreted with the help of the graph of this function. In Fig. 18 we can clearly identify some varying domains of the function.

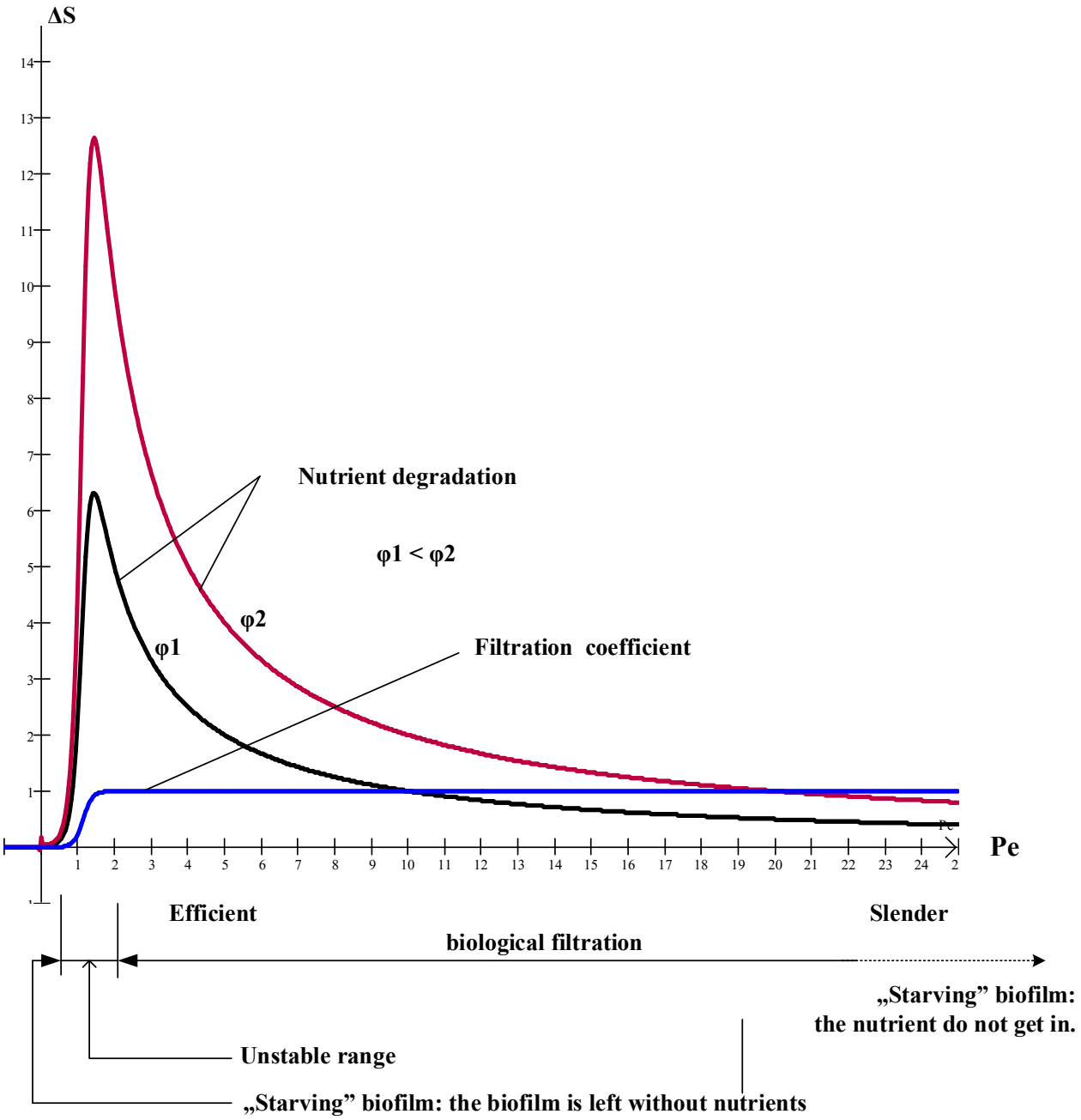


Fig. 18 Nutrient degradation curves.

In this nutrient degradation graph four distinct domains can be identified:

- $Pe < 1$ represents the case of the "starving" biofilm, where the biofilm is not reached by enough nutrients, although there is enough time available for the diffusion to be completed,
- $Pe \sim 1$ represents the unstable range, where even the least significant change can cause a sudden turnback,
- $Pe > 1$ represents the stable and effective range, and
- $Pe >> 1$ represents another low efficiency range, again described as the case of the "starving" biofilm. Nevertheless, in this range there is an other reason why the nutrient cannot penetrate the biofilm, namely that the velocity is too high, causing nutrient and biofilm to pass by each other.

All of the above allows us to draw the most important conclusion regarding the theory of biological filtration:

> **Biological filters are at their most effective in the *Pe* > 1 domain.** While dimensioning and scaling the plant, the *Pe*-number must be positioned in a way that it falls within this range.
>
> The efficiency of the process can be further increased by adjusting the "climatic" conditions. Regarding the function this would mean "conversion" to a curve with a higher φ parameter value.

2.8 Interrelated Theories

There exists a formal link between the Michaelis-Menten and the Monod kinetic models. With both kinetics, enzymes play a decisive role. However, while the enzyme acts as catalyst in the former, it functions as structural element of the cell in the latter. The reaction rate and the Monod-factor are both dependent on the substrate content, and the nature of functional relationships can, in both cases, be described with the help of the half-saturation constant.

Likewise, the connection between Monod-kinetics and biological filtration tends to be formal as well. One of the similarities here is provided by the participation of bacteria, described in the one case through their reproduction rate and, in the other, through the surface demand for their colonization. Another similarity can be seen in the formula of the Logistic Function, which similarity — along with the above — represents the connection between the two models.

However, it is by no means enough to recognize the interrelated nature of the above theories. Although each one is formed around a different subject matter, involving different places of occurrence and key parameters, together they appear to form a whole new system of interrelated models. Fig. 19 illustrates the way they are interrelated with one another.

At the same time, they also represent a sequence of stages through which we can get from the cells through the microbes to the processes described by the theory of biological filtration or, in end effect, to the purification of water.

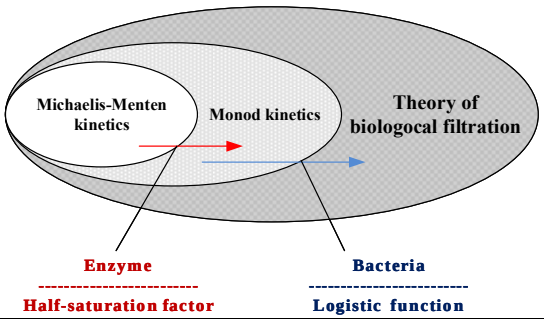
3. Conclusion and Outlook

As Károly Simonyi says in A Cultural History of Physics:

> Newtons laws and Maxwell's equations are axioms. It is not because they are so apparent that they are true, but because the conclusions relying on them equal reality.

Although, regarding their importance, the theory on biological filtration put forward above falls far behind the theories represented by Newton's and Maxwell's equations, the message expressed in this quote does have a certain relevance to our case. The nutrient degradation model is only valuable in the case if it can be used to explain relevant real-life phenomena and forecast some aspects or outcome of the observed processes. A division of the process into **physical and biochemical sub-processes** enables us to provide more reliable and extensive answers to a number of questions than made possible by relying on our earlier approach. With no attempt at providing a full list, in the following we will simply enumerate some of these questions,

leaving it to the reader to infer the answers from the axioms put forward in this paper.



Process	Subject	Place of occurrence	Key parameter
Michaelis-Menten kinetics	Cell metabolism	Cell	Reaction rate
Monod kinetics	Microbial reproduction	Biofilm	Relative growth factor
Theory of biological filtration	Biological filtration	Biological reactor	Filtering factor

Fig. 19 Interrelated processes.

- Why can bank filtration remain efficient in winter, whereas other wastewater treatment technologies tend to lose their efficiency below ca. 10°C?
- What are the similarities and differences between the various wastewater treatment technologies (e.g., activated sludge technology, trickling filter technology, SBR technology, the Nereda treatment technology, etc.)?
- Why are the structures used with the Nereda technology much smaller than the conventional ones?
- Is there a point in even considering the aspects of an immobile biofilm or immobile activated sludge?
- Should membranes be regarded as small-pore-size filters, or rather as biofilm carrier surfaces hosting living organisms?
- Is it true that a bio-module configuration – with plant roots in the wastewater treatment reactors – can considerably increase filtration efficiency?
- Is it still a valid statement that wastewater treatment reactors must be kept homogenous and thoroughly agitated, or is there a more precise definition to describe the requirement?
- Can *residence time* and *sludge age* still be regarded as the most suitable variables to describe time-related aspects?
- Should the sludge in the digester be homogenized or, rather, disintegrated? What exactly is happening in there?
- Why do we need to agitate and heat the sludge in the digester?

In addition to finding an answer to the arising questions, it is equally important to outline the tasks to be performed in the future and to determine the direction of further research. Louis-Claude Vincent conducted extensive research into water-borne diseases and the pathogenic microbes causing them. During his research, he aimed to deprive these microbes from their vital conditions. The Vincent diagram maps out the living area of pathogenic microbes in a *pH-rH* coordinate system [13].

During his fight against pathogenic micro-organisms, Vincent put forward the following theorem:

> Deprive the disease from its vital conditions, and the disease shall cease to exist!

In other words, all we need to do is change the environment determined by *pH* and *rH* so that the conditions required by the pathogenic microbes to reproduce shall not be fulfilled.

The connection existing between the *Ne* factor used in Eq. (1) and the *pH* and *rH* values of the Vincent diagram can be described as follows:

$$rH = \frac{E_h}{RT}\frac{2F}{g} + 2\ pH = \frac{1}{g}Ne + 2pH \tag{7}$$

where

rH: has a value varying between 0-42, helps in the classification of microbiological activity;

g: is the conversion factor for logarithm bases, $g = 1/lg(e) = 2{,}303$

pH: is a dimensionless parameter specifying the acidity or basicity of the substance, has a value varying between 0 … 14.

Unlike in the case of disinfection, with biological water filtration bacterial life is not "hunted down". Our purpose is quite the opposite: we strive to support bacteria, thus enabling them to perform their jobs with the highest possible efficiency. To facilitate this, climatic conditions must be specified in a way that they fall within the range favouring bacterial activity. In addition, as nutrient degrading microbes are not yet represented in Fig. 20, mapping them out could be one of the tasks outlined for the future.

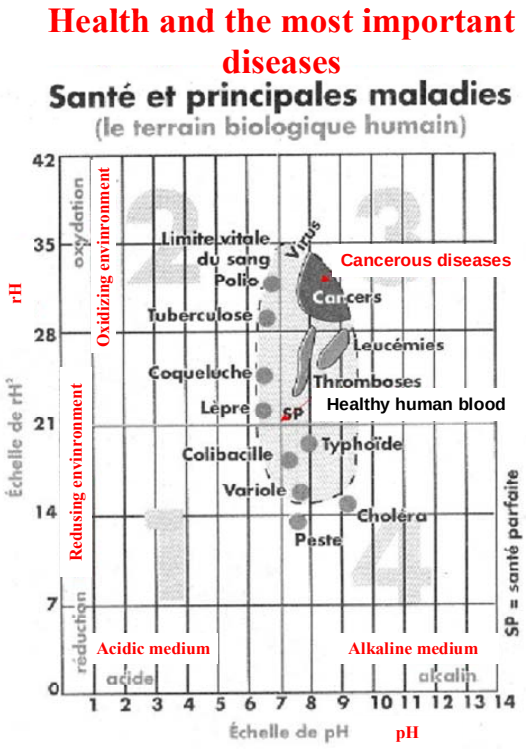


Fig. 20 Vincent diagram for diseases.

References

[1] B. Tolnai, Bevezetés a biológiai szűrés modellezésébe és méretezésébe (Introduction into the modeling and dimensioning of biological filtration) Patent application, National Patent Office of Hungary, Number: P0800635/2008.

[2] B. Tolnai, Chapters from the topic of biological filtration and application, in: *4th International Symposium Re-Water Braunschweig*, 6-7 Nov., 2013.

[3] B. Tolnai, Sewage utilisation instead of sewage treatment, In: *5th International Symposium Re-Water Braunschweig*, 2-3 Nov., 2015.

[4] Th. Szirtes, *Applied Dimensional Analysis and Modeling*, Elsevier Butterworth-Heinemann Ontairo, Canada, 2007.

[5] K. Sydsæter and P. I. *Hammond, Matematika közgazdászoknak (Matematics for Economic Analysis)*, Chapter 8.4.Aula Kiadó, Budapest, 2000.

[6] Th. Junker, *A biológia története (Geschichte der Biologie, Die Wissenschaft vom Leben)*, Corvina Kiadó, Budapest 2007.

[7] P. Eckert and R. Irmscher, Over 130 years of experience with Riverbank Filtration, *Düsseldorf Journal of Water Supply* 4 (2006).

[8] I. Jekel and S. Grünheid, Ist die Uferfiltration eine effektive Barriere gegen organische Substanzen und Arzneimittelrückstände? *GWF Wasser-Abwasser* 148 (2007) Nr. 10.

[9] KKFT BME A biológiai rendszerek kinetikája (The kinetics of biological systems), 2000, available online at: http://kkft.bme.hu/sites/default/files/Vebi_Biomernoki.pdf.

[10] Montana University, Biofilms in Drinking Water Treatment and Distribution, 2007, available online at: http://www.erc.montana.edu/biofilmbook.

[11] E. Szűcs, *Hasonlóság és model (Similarity and Model)*, Műszaki Könyvkiadó, Budapest, 1972.

[12] Water Environment Federation Biofilm Reactors WEF Manual of Practice No. 35. Mc Graw Hill, 2010.

[13] J. Országh, available online at: http://www.eautarcie.com.