

## The Determining Torsional Vibration Damping Coefficients Algorithm for Computing Marine Shafting's Vibrations

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**Abstract:** The most difficulty in the computations of ship shafting torsional vibrations is to determine such torsional vibration damping coefficients as those in main engines and shafting structures, or those of hydraulic resistances on propellers. The difficulties are due to the complex nature of hydrodynamical processes, that it is difficult to come up with an analytical method to determine these quantities. Even direct measurements of such quantities are difficult to make in practice.

Currently in Vietnam, these data usually are given by the manufacturers that doesn't make sure about the accuracy, or they are calculated based on the semi-empirical formulae those are old and may be able not appropriate for the modern types of engines and equipments. Therefore, the aim of the author is to determine these quantities from results of torsional vibration measurements during the new built ship tests. Basing on the analysis of results obtained for shafting's types with different features (for example, characteristic types of engines, propellers, shaft materials ...) and combining with the theoretical analysis we may generalize calculations of the above quantities. It allows to obtain the new calculating formulae for these quantities more certain and convenient for basic design calculation of ship shafting.

The paper presents a theoretical basis and algorithms of determining the torsional vibration damping coefficients of the engine structure, shaft material and hydrodynamic damping coefficient to propellers from results of vibration measurements. By programming with the help of Symbolic Math Toolbox package in Matlab software we will establish an analytical equations for amplitudes of elastic torque  $M(k\omega)$  caused by torsional vibrations in an arbitrary shaft section (where perform measurements conveniently). This equation will contain  $m$  unknown variables supposed to be unknown damping coefficients for each given shafting with given sizes, mass parameters and the finite number of masses. Therefore, to identify these  $m$  unknown variables, we must have at least  $m$  values of torque amplitudes corresponding to  $m$  values of  $k\omega$  measured in a position of shafting to make a system of  $m$  equations. However, the system of equations is not linear so to facilitate the calculations we need to have more than  $m$  values of the measured amplitudes. The program will indicate how many values of the measured amplitudes should be obtained, and after having the measurement results, it will determine the unknown damping coefficients.

To illustrate, the article presents the example calculation for a ship of series B 170-V designed by Polish Architecture Institute and built in Ha Long shipyard. The data on the shafting and the results of vibration

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measurements obtained from the documents submitted by Ha Long shipyard and the Designer to GL registry for approval. The obtained results of calculation will explain the reason for deviations between real vibrations and computed vibrations or show the appropriateness of assumptions about the damping coefficients used in computations of torsional vibrations. Basing on this profile, we can be able to improve the vibration calculation model.

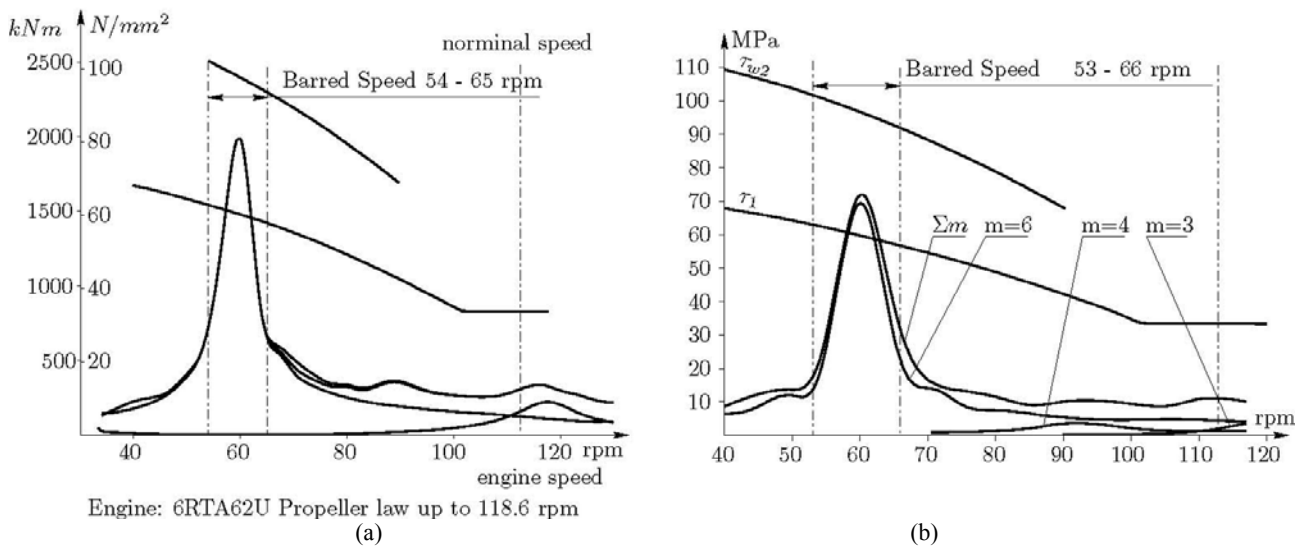
**Key words:** torsional vibrations, damping coefficient, relative and absolute damping coefficients, elastic torque.

### 1. Introduction

As we know, the kinds of resistances acting on the shafting's vibrations in general are not constant, but non-linearly depend on amplitudes and velocities of vibrations. For example, the resistance in the shaft's material (relative resistance) or hydraulic resistances on propellers (absolute resistance) is the exponential functions of torsional deformation amplitudes or torsional vibration amplitudes of propellers (for example, the resistant work to propellers per a vibration cycle is proportional to square of the vibration amplitude (Đặng Hộ, 1986)). Therefore here we should solve systems of nonlinear differential equations. It is the reason why such complicated resistant models are used to calculate only resonant vibrations, assuming that forms of forced vibrations are similar to forms of free vibrations (Đặng Hộ, 1986; Nguyễn Mạnh Thường, 2011).

But currently, many Design Agencies and Registries have accept assumptions, supposing these damping coefficients are constants and resistant torques proportional to the speed of deformations. The damping coefficients of the water on propellers accord to the Archer formula.

Vibatory Torque and Stress Intermediate Shaft



**Figure 1 (a) Stress Amplitude/Rotation Speed at Intermediate Shaft Calculated by the Polish Designers; (b) Stress Amplitude/Rotation Speed at Intermediate Shaft Measured**

Figures 1a and 1b show calculated by the Polish design agency and measured torques of elastic deformations at intermediate shaft of B-170-V ship and Figure 2 shows the calculation results of the authors of this paper. As seen on the pictures, the amplitude-frequency curves of calculations and of actual measurements are rather

identical. Differences in quantities are interpreted as errors caused by using simplifying assumptions about resistances.

In spite of the fact that there are differences between computations and measurements, we may see that, computation results with the above simplifying approximate assumptions meet the purposes of torsional vibration calculations for basic design stage and satisfies the requirements of the Registry for the torsion vibration calculations. But essential problem posed now is how to determine these damping coefficients. How do we design without data given by manufacturers? What is based on to confirm results of design vibration calculations be reliable?

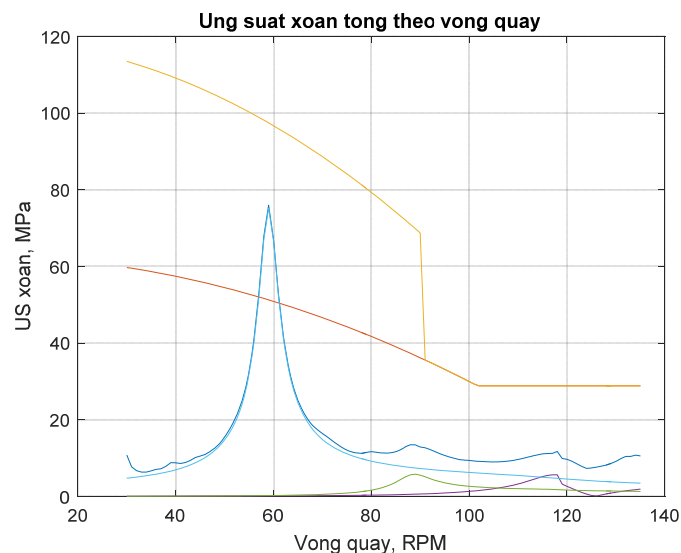


Figure 2 Stress Amplitude/ Rotation Speed at Intermediate Shaft Calculated by the Author

We may infer that, the determination of the above damping coefficients still is a problem to be solved even for the world developed shipbuilding industry because the foreign registry offices still require testing torsional vibration measurements for new designed ships.

This paper will present algorithms determining damping coefficients having had the torsional measurement results when testing new build for the use in the future to study these types of resistances.

Indeed, after determining the damping coefficients for many different ship shaftings, we will have more experience and understanding of the factors causing resistances that may help us to establish semi-empirical formulas to determine the coefficients of resistances in engines and in the shaft material or resistances to propellers by water ... to serve torsional vibration calculation in design stage.

## 2. Establish Algorithms Determining the Damping Coefficients

Considering shafting being equivalent to a torsional vibration system of  $n$  discrete masses, linking together by elastic elements without weight as shown in Figure 3 (Đặng Hộ, 1986). The vibration driving forces are forces from cylinders acting on engine crank shafts and of water on propeller. The main forms of resistance included are those in the structure of piston-cylinder/connecting rod/crank shaft, and those in shaft material and of water on propellers. Designating angle of torsion deformation of  $i$ -th mass as  $\varphi_i$ , forcing torque on that mass as  $M_i$ , inertial mass moment of the  $i$ -th mass as  $I_i$ , absolute damping coefficient-  $a_i$ , relative damping coefficient between two

masses  $i$  and  $i+1$  as  $b_{i,i+1}$  and stiffness between them-  $K_{i,i+1}$ , we have dynamical equation of  $i$ -th mass as (Nguyễn Mạnh Thường, 2011):

$$-b_{i-1,i}\dot{\varphi}_{i-1} - K_{i-1}\varphi_{i-1} + I_i\ddot{\varphi}_i + (a_i + b_{i-1,i} + b_{i,i+1})\dot{\varphi}_i + (K_{i-1,i} + K_{i,i+1})\varphi_i - b_{i,i+1}\dot{\varphi}_{i+1} - K_{i,i+1}\varphi_{i+1} = M_i \quad (1)$$

where:  $i = 1, \dots, n$ .

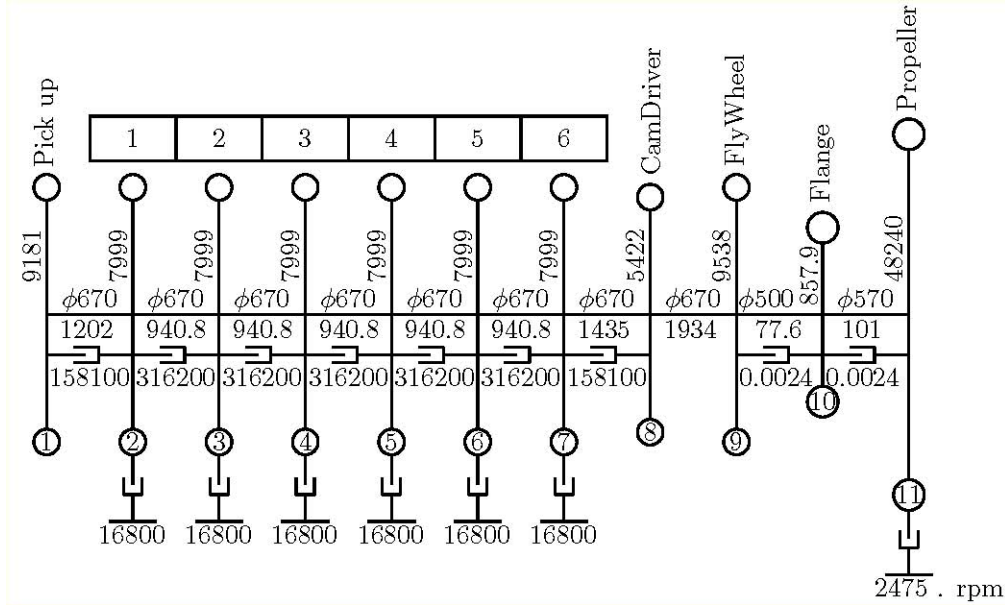


Figure 3 A Torsional Vibration Model of B\_170 V, Main Engine - 6 RTA 62 U

For stable vibrations, torsional angle of  $i$ -th mass is equal sum of harmonic vibrations:

$$\varphi_i = \sum_1^m (S_{i,k} \sin k\omega t + C_{i,k} \cos k\omega t), \quad (2)$$

and the exciting moments, being cyclic, can be presented by Fourier's series:

$$M_i = \sum_1^m M_{i,k} \sin(k\omega t + \beta_i) = \sum_1^m (M_{i,k}^s \sin k\omega t + M_{i,k}^c \cos k\omega t) \quad (3)$$

Replacing (2) and (3) into (1) we will get an equation system for each harmonic order  $k$ , after equating coefficient of  $\sin$  and  $\cos$  we will get a system of  $2 \times n$  algebraic equations as:

$$F_k A_k = M_k, \quad (4)$$

where:

$$M_k = [M_{1,k}^s, M_{2,k}^s, \dots, M_{n,k}^s, M_{1,k}^c, M_{2,k}^c, \dots, M_{n,k}^c]^T \text{ — column vector of exciting torque amplitudes at harmonic}$$

order  $k$ ;

$$A_k = [S_{1,k}, S_{2,k}, \dots, S_{n,k}, C_{1,k}, C_{2,k}, \dots, C_{n,k}]^T \text{ — column vector of torsion angle amplitudes according to}$$

harmonic order  $k$ ;

$F_k = [f_{ij}]$  – square matrix  $2 \times n$ , its elements  $f_{ij}$  are functions of  $k\omega$ ; stiffnesses of shafts  $K_{i,i+1}$ ; inertial moment  $I_i$  and of damping coefficients  $a_i, b_{i,i+1}$  (Nguyễn Mạnh Thường, 2014).

In calculations of forced vibrations, the elements  $f_{ij}$  of  $F_k$  are determined when damping coefficients are known, and then torsional vibration amplitudes of masses due to harmonic  $k$  are computed by:

$$A_k = F_k^{-1}M_k \quad (5)$$

Then elastic torsion torque amplitude at order  $k$  in shaft between masses  $i$ -th and  $(i+1)$ -th is calculated by:

$$E_{i,i+1}^k = K_{i,i+1} \sqrt{(S_{i,k} - S_{i+1,k})^2 + (C_{i,k} - C_{i+1,k})^2} = K_{i,i+1} \Delta\varphi_{i,i+1} \quad (6)$$

If supposing we have measured the elastic torsional torques in a shaft  $E_{i,i+1}$  (see Figure 4), then according to (6) we could determine the amplitudes of relative torsion deformations  $\Delta\varphi_{i,i+1}$  between two masses  $i$  and  $i+1$ :

$$\Delta\varphi_{i,i+1} = E_{i,i+1}^k / K_{i,i+1} \quad (7)$$

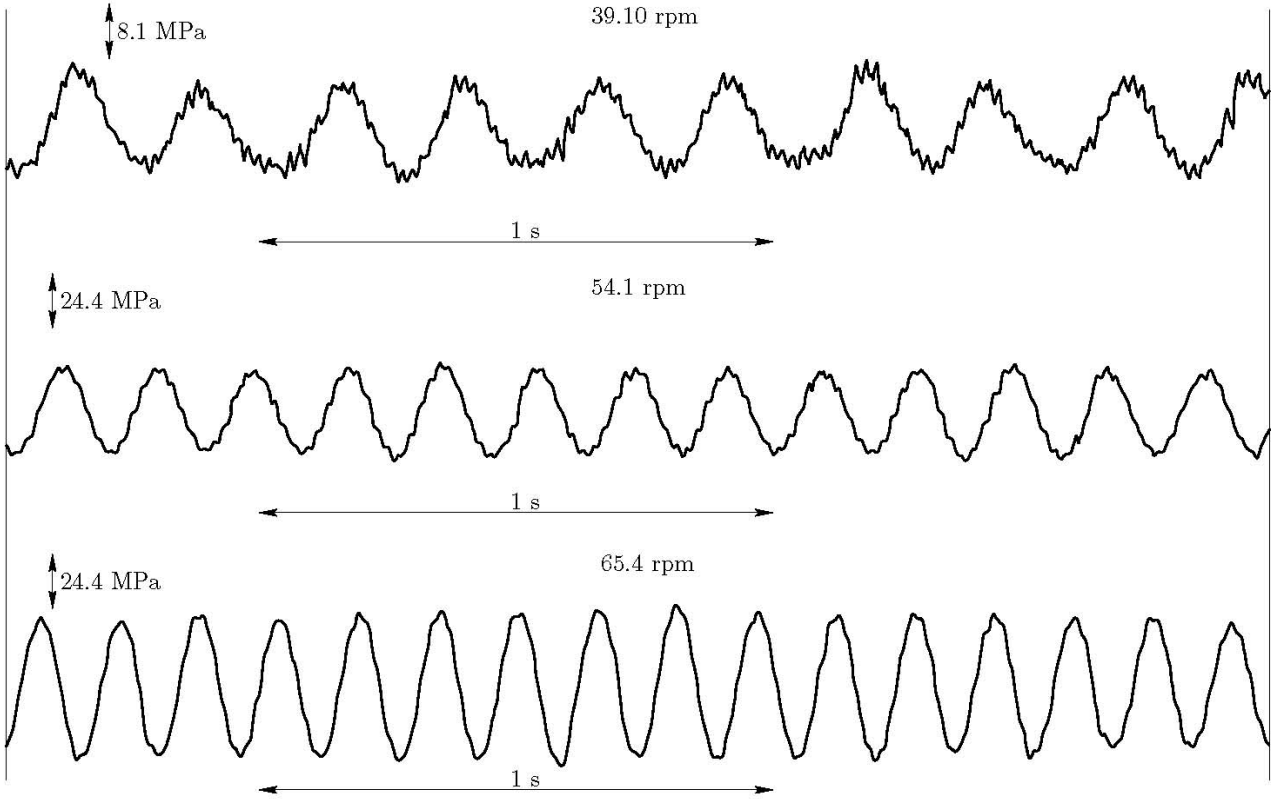


Figure 4 Stress/Time at the Intermediate Shaft at Different Rotation Speeds

On the other side,  $\Delta\varphi_{i,i+1}$  can be calculated by equation (5) as following:

$$\{[\Phi_{i+1,1...2n}] - [\Phi_{i,1...2n}]\} \times M_k = (S_{i+1,k} - S_{i,k}) \cdot \det(F_k)$$

$$\{[\Phi_{n+i+1,1...2n}] - [\Phi_{n+i,1...2n}]\} \times M_k = (C_{i+1,k} - C_{i,k}) \cdot \det(F_k)$$

Therefore:

$$\Delta\varphi_{i,i+1}^2 \det^2(F_k) = \left\{ \left( [\Phi_{i+1,1...2n}] - [\Phi_{i,1...2n}] \right)^2 + \left( [\Phi_{n+i+1,1...2n}] - [\Phi_{n+i,1...2n}] \right)^2 \right\} \cdot M_k^2, \quad (8)$$

where:  $[\Phi_{i+1,1...2n}]$ ;  $[\Phi_{i,1...2n}]$ ;  $[\Phi_{n+i+1,1...2n}]$ ;  $[\Phi_{n+i,1...2n}]$  are lines  $i$ ,  $i+1$  and  $n+i+1$ ,  $n+i$  of matrix  $[\Phi]$  with elements

determined as:  $\Phi_{i,j} = D_{j,i}$  - where  $D_{j,i}$  - algebraic complement of the element  $(j,i)$  of  $F_k$ .  $\Delta\phi_{i,i+1}$  can be calculated according to equation (7) if the elastic deformation torque is measured.

Thus, suppose there are  $m$  values of damping coefficients  $a_j$  and  $b_{j,j+1}$  unknown and should be defined. Then equation (8) will contain  $m$  unknown variables (they are damping coefficients in  $F_k$ ). In principle, in order to determine the  $m$  these variables, we should get  $m$  values of elastic torsional torque in shaft between masses  $i, i+1$  (see Figure 4) in order to make  $m$  algebraic equations to determine  $m$  unknown values of  $a_j$  and  $b_{j,j+1}$ . However, the system of equations obtained is the type of multi-hidden variable and high order, so that to be able to solve it, we should make more than  $m$  equations to obtain a system of linear equations with the new hidden variables, those are combinations of productions of unknown as:  $x_i = a_1^{m1} \cdot a_2^{m2} \dots b_1^{n1} \cdot b_2^{n2} \dots$

To illustrate, here we consider the ships of series B 170 V. Its torsional vibration model is as shown in Figure 3 having the following characteristic parameters.

- The number of concentrated masses:  $n = 11$ ;

- The inertial moments of concentrated masses ( $\text{kgm}^2$ ):

$$I_i = [9181.0 \ 7999.0 \ 7999.0 \ 7999.0 \ 7999.0 \ 7999.0 \ 7999.0 \ 5422.0 \ 9538.3 \ 857.9 \ 48240.1];$$

- The stiffnesses of shafts ( $\text{MNm/rad}$ )

$$K_i = [1201.92 \ 940.82 \ 940.82 \ 940.82 \ 940.82 \ 940.82 \ 1434.93 \ 934.24 \ 77.16 \ 101.04];$$

- Diameters of shafts (mm):

$$D_i = [670 \ 670 \ 670 \ 670 \ 670 \ 670 \ 670 \ 670 \ 500 \ 570];$$

Designating unknown damping coefficients  $a_i$  and  $b_{i,i+1}$ , with symbols  $s, z, f, y, r$  and place them into equation (1), we get matrix  $F_k(22,22)$  for each value of  $kw$  ( $k$ - harmonic order,  $w$ - rotation speed of propeller), having form:

$$F_k = [1201920 - 9.18 \cdot kw^2, -1201920, 0, \dots, kw \cdot r, kw \cdot r, \dots, 0]$$

$$[-1201920, 2142740 - 7.99 \cdot kw^2, -940820, 0, \dots, 0, kw \cdot r, -kw \cdot z, kw \cdot s, \dots, 0]$$

$$[0, -940820, 1881640 - 7.99 \cdot kw^2, -940820, \dots, kw \cdot s, -f \cdot kw, kw \cdot s, \dots, 0]$$

.....

$$[-kw \cdot r, kw \cdot z, -kw \cdot s, \dots, -1201920, 2142740 - 7.99 \cdot kw^2, -940820, \dots, 0]$$

.....

$$[0, 0, \dots, -77160, 178200 - 0.85 \cdot kw^2, -101040]$$

$$[0, \dots, kw \cdot y, \dots, 0, -101040, 101040 - 4.82 \cdot kw^2]$$

To determine the determinant of matrix  $F_k = \det(F_k)$  and the algebraic complements  $D_{i,j}$  can be done with using the Symbolic Math Toolbox package of Matlab software by the command “ $\det(F_k)$ ” if the matrix size is small enough (depending on the number of non-zero elements), the result is a formula in symbolic form on the screen, for example:

$$\text{ans} = 2871048086689088639517089901.315 \cdot kw^{14} \cdot r^3 \cdot s^4 \cdot z + 218063572479354423019604109325.3 \cdot kw^{16} \cdot r^2 \cdot s^3 \cdot z + \dots + 12883815368824962090418376$$

Finally, with each value of  $kw$ , after having computed all the algebraic complements  $D_{i,j}$  of elements in the interesting lines, replacing them into equation (8), supposing we get an equation as:

$$C_1 f^{15} r^2 z + \dots + C_i z^7 s^4 \dots + C_{p-2} s + C_{p-1} y + C_p r = C_0 \tag{9}$$

There are  $p$  constants  $C_i$  in the left side of the equation (9), according to  $k$  and angular speed  $w$  of the propeller shaft. Here, to determine  $r, f, z, \dots$ , we should obtain  $p$  values of torsion torque amplitudes according to  $p$  value of  $kw$  in order to make  $p$  equations (9) and a system of  $p$  linear equations with  $p$  hidden:  $x_1 = r, x_2 = f, x_3 = z, \dots, x_i = f^m r^n \dots z, \dots$ . Resolving this system we would identify the hidden  $r, f, s, \dots$ .

If the computing is performed correctly and with inputs being the elastic torque values obtained by calculations as described above (assuming the damping coefficients are constant), then the result must match, it means, for example:

$$\text{if: } x_1 = r, x_2 = f, x_3 = z, x_i = f^m r^n z,$$

$$\text{then: } x_i = x_2^m x_1^n x_3.$$

Being sure it correct, take values of elastic torsional torques measured practically to put into equation (9) to calculate. If  $x_i \neq x_2^m x_1^n x_3$ , for example, depending on differences we may assess the suitability of the used assumptions about resistances.

However, in the example case, unfortunately we cannot directly use the command  $\det(F_k)$ . It would cause errors "Out of Memory" or the text be too long, we would get a report on screens, for example:

$$\text{ans} = 2871048086689088639517089901.315 * kw^{14} * r^3 * s^4 * z + \dots$$

$$\dots + .2218063572479354423019604109325.3 * kw^{16} * r^2 * s^3 * z + 12883815368824962090418376 \dots$$

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To overcome this problem, we should divide the calculation procedure into some steps:

**Step 1:** replace the original terms of the first matrix by new symbols (to get more compact form), for example:

$$I1 = -I1 * kw^2 + K1; r = kw * r; I2 = -Ic * kw^2 + K1 + Kx; z = kw * z; s = kw * s; I3 = -Ic * kw^2 + 2 * Kx;$$

$$I4 = -Ic * kw^2 + 2 * Kx; I5 = -Ic * kw^2 + K3 + Kx; I6 = -I3 * kw^2 + K3 + K4 = I6; I7 = -I4 * kw^2 + K4 + K5;$$

$$I8 = -I5 * kw^2 + K5 + K6; I9 = -Ic * kw^2 + K6,$$

to obtain matrix  $F_k$  looking as:

$$F_k = [I1, -K1, 0, \dots, r, r, \dots, 0];$$

$$[-K1, I2, -Kx, 0, 0, 0, 0, 0, 0, 0, r, -z, s, 0, 0, 0, 0, 0, 0, 0];$$

$$\dots$$

$$[r, -r, 0, \dots, 0, I1, -K1, 0, \dots, 0];$$

$$\dots$$

$$[0, \dots, 0, K5, I8, -K6];$$

$$[0, \dots, 0, -K6, I9];$$

**Step 2:** use the comand "det" for a matrix of smaller size, for example:  $D10 = \det(F_k(1:10,1:10))$ . The obtained results look like that:

$$D10 = - I3 * I4^3 * K1^2 - I1 * I4^3 * Kx^2 + I1 * I2 * I3 * I4^3 + I4^2 * K1^2 * Kx^2 - I1 * I2 * I4^2 * Kx^2 + 2 * I3 * I4 * K1^2 * Kx^2 + 2 * I1 * I4 * Kx^4 - 2 * I1 * I2 * I3 * I4 * Kx^2 - K1^2 * Kx^4 + I1 * I2 * Kx^4$$

**Step 3:** transform  $F_k$  to  $F_k^{-1}$  by Gause method so that  $F_k^{-1}(1:10,1:10)$  be a triangle matrix;

**Step 4:** replace elements of  $F_k^{-1}(11:22,11:22)$  with another symbols to get a new matrix of  $12 \times 12$ :

$$B_k = [b1\_1, b1\_2, b1\_3, b1\_4, b1\_5, b1\_6, b1\_7, b1\_8, b1\_9, 0, 0, b1\_12]$$

$$[b2\_1, b2\_2, b2\_3, b2\_4, b2\_5, b2\_6, b2\_7, b2\_8, b2\_9, 0, 0, 0]$$

$$[b3\_1, b3\_2, b3\_3, b3\_4, b3\_5, b3\_6, b3\_7, b3\_8, b3\_9, 0, 0, 0]$$

$$[b4\_1, b4\_2, b4\_3, b4\_4, b4\_5, b4\_6, b4\_7, b4\_8, b4\_9, 0, 0, 0]$$

[b5\_1, b5\_5, b5\_3, b5\_4, b5\_5, b5\_6, b5\_7, b5\_8, b5\_9, 0, 0, 0]  
[b6\_1, b6\_2, b6\_3, b6\_4, b6\_5, b6\_6, b6\_7, b6\_8, b6\_9, 0, 0, 0]  
[b7\_1, b7\_2, b7\_3, b7\_4, b7\_5, b7\_6, b7\_7, b7\_8, b7\_9, 0, 0, 0]  
[b8\_1, b8\_2, b8\_3, b8\_4, b8\_5, b8\_6, b8\_7, b8\_8, b8\_9, 0, 0, 0]  
[b9\_1, b9\_2, b9\_3, b9\_4, b9\_5, b9\_6, b9\_7, b9\_8, b9\_9, b9\_10, 0, 0]  
[0, 0, 0, 0, 0, 0, 0, 0, b10\_9, b10\_10, b10\_11, 0]  
[0, ... 0, 0, 0, ..... b11\_10, b11\_11, b11\_12]  
[b12\_1, 0, ..... ..0, b12\_11, b12\_12]

**Step 5:** repeat step 1), for example:  $\det(B_k(1:6,1:6))$ , and then do the same as in step 2)...

The purpose of this algorithm is to divide the final formula of  $\det(F_k)$  in to shorter terms so that they can be displayed on the screen.

### 3. Conclusion

Thus, the above presented algorithms allow determining the damping coefficients if having measured torsion elastic torque amplitudes at any shaft section. Numbers of measurement results should obtain will depend on number of masses.

Now the paper's author has just got the formula for  $\det(F_k)$  in a form of string of characters written with syntax errors for Matlab to be corrected and the final results haven't obtained yet because of that it takes long time, but when getting it we can use to calculate for other systems with smaller number of discrete masses.

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