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Frequency Analysis of Aftershock Using Possion Distribution: A Case Study of Aftershock in Nabire, Papua in 2014 in Every 6-Hour Interval

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Abstract: There have been many researchers using Poisson Distribution to analyze the parameters of earthquake magnitudes data. The results of parameter estimation of the magnitudes are then illustrated in some curves to be analyzed. In this article, there will be a discussion in analyzing the data of earthquake frequency using Poisson Distribution by estimating the parameters using Maximum Likelihood Estimation (MLE) method. Then, the parameters are used to determine the magnitudes and the energy of the earthquake. The case study in this research used the data of aftershock frequency in Nabire District in Papua in 2004 for every 6-hour interval. After conducting the compliance test using Chi Square test, it obtained that the data followed the pattern of Poisson Distribution. It is clearly seen that the period of 03.00 a.m.–09.00 a.m. has the biggest frequency of earthquake compared with other hours in which the parameter λ = 42.97, mean = 42.97, magnitude (M) = 6.03 richter scale, energy (E) = 75.55. For the description: this earthquake was felt by the people driving a car, and the buildings with bad constructions would be collapsed as well as for their chimneys. Thus, this research is expected to be able to reduce the number of victims as well as to minimize the loss caused by the earthquake.

Key words: possion distribution, MLE, Newton Raphson, Chi Square Test, magnitude, energy, Marcelli Intensity.

1. Introduction

Earthquake is a shock or shaking on the earth's surface caused by a sudden release of endogenous energy that creates seismic waves. Indonesia is one of the vulnerable countries to earthquakes. It is because Indonesia is located in a meeting point of three tectonic plates. This condition makes the regions of Indonesia as the active tectonic regions with high seismicity including Papua and West Papua regions. This condition results in many earthquakes occurred in those regions.

For example, in 2004, a 7.2 richter scale earthquake hit Nabire, Papua. It resulted in paralysis/isolation in the city due to the disconnected electrical and phone lines even damage in the Nabire airport. It needs special attention both from the government and related institutions, so that there will be a probability of occurrence in the future. By understanding the characteristics and history of disasters in the past, including the last occurrence and the

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estimation of repeating period of occurrence, can minimize the loss.

If the occurrences of big earthquakes in the past in a certain region are studied carefully, the data can be used to estimate or predict the similar occurrences in the future. Thus, it can also be used for mitigation of earthquake in the future.

2. Basic Theory

2.1 Poisson Distribution (Walpole, 1995)

A T-discrete random variable is said to have a Poisson distribution with a parameter $\lambda > 0$ having a pdf-discrete in the form of:

$$P(t;\lambda) = \frac{e^{-\lambda}\lambda^{t}}{t!} \tag{1}$$

So that the survival form of Poisson Distribution is as follow:

$$S(t) = 1 - \sum_{j=0}^{t-1} \frac{\lambda^{j}}{j!} e^{-\lambda} \tag{2}$$

Because: $\mathrm{F(t)} = 1 - \mathrm{S(t)}$

Then,

$$\mathrm{F(t)} = 1 - \left(1 - \sum_{j=0}^{t-1} \frac{\lambda^{j}}{j!} e^{-\lambda}\right) = \sum_{j=0}^{t-1} \frac{\lambda^{j}}{j!} e^{-\lambda} \tag{3}$$

The hazard function obtained $h(t)$ as:

$$h(t) = \frac{f(t)}{S(t)} = \frac{\frac{e^{-\lambda}\lambda^{t}}{t!}}{1-\sum_{j=0}^{t-1}\frac{\lambda^{j}}{j!}e^{-\lambda}} \tag{4}$$

2.2 Likelihood Function (Bain & Engelhardt, 1992)

If $T_1, T_2, \ldots, T_n$ states the random variables are mutually independent with the probability density function $f(t_i\,;\theta)$, where θ are c, b that are the parameters that will be estimated. If L is the joint probability mass function/function joint opportunities of $T_1, T_2, \ldots, T_n$ that is seen as the function of θ, its likelihood function is shown by:

$$L(\theta) = \prod_{1}^{n} f(t_i\,;\theta) \tag{5}$$

θ value that maximizes the L(θ) will also maximize the log likelihood $(lnL(\theta) = l(\theta))$. The $\hat{\theta}$ is obtained by conducting the following steps:

(1) The $\hat{\theta}$ value is obtained from the first derivative: $\frac{\partial l(\theta)}{\partial\theta} = 0$

(2) The $\hat{\theta}$ value is said to maximize $l(\theta)$ if $\left.\frac{\partial^2 l(\theta)}{\partial\theta^2}\right|_{\theta=\hat{\theta}} < 0$ (negative definite)

2.3 Mean and Variance of Poisson Distribution

For the expected value T is symbolized by $E(T)$ and variance T is symbolized by $Var(T)$ defined as follows:

$$E(T) = \int_{-\infty}^{\infty} t\, f(t)dt. \tag{6}$$

$$Var\,(T) = E(T^2) - \{E(T)\}^2 \tag{7}$$

According to equations (6) and (7), the expected value of T for Poisson Distribution is:

$$E(T) = Var(T) = \lambda$$

2.4 Magnitude to Measure Earthquake

The relationship between frequency and magnitude proposed by Gutenberg-Richter is stated in the following formulation:

$$Log\ N(M) = a - bM \tag{8}$$

For Indonesia (a = 7.30 and b = 0.94), then $LogN(M)$ = 7.30-0.94M.

While the relationship between Richter Scale and the magnitude of energy released at the occurrence of earthquake can be stated in an equation as follow:

$$LogE = 11.4 + 1.5M \tag{9}$$

3. Estimation of Poisson Distribution Parameters

3.1 Survival Function of Poisson Distribution

A T-discrete random variable is said to have Poisson Distribution with parameter λ > 0 having a pdf-discrete in the form of:

$$P(t;\lambda) = \frac{e^{-\lambda}\lambda^t}{t!} \tag{10}$$

So that the survival form of Poisson Distribution is as follow:

$$S(t) = 1 - \sum_{j=0}^{t-1} \frac{\lambda^j}{j!} e^{-\lambda} \tag{11}$$

Because: $F(t) = 1 - S(t)$

So that $F(t)$ is:

$$F(t) = \sum_{j=0}^{t-1} \frac{\lambda^j}{j!} e^{-\lambda} \tag{12}$$

Then, it will find the mean and variance of Poisson Distribution:

$$E(T) = Var(T) = \lambda \tag{13}$$

3.2 Estimation of Poisson Distribution Using MLE

The *likelihood* function of Poisson Distribution with parameter λ can be described as follows:

$$L(\theta) = \prod_{1}^{n} f(t_i\,;\theta)$$

$$L(t_1, \dots, t_n; \lambda) = f(t_1;\lambda) \cdot \dots \cdot f(t_n;\lambda)$$

$$l = \ln L = ln\left[\frac{1}{t_1!\,t_2!\dots t_n!}\frac{e^{-n\lambda}\lambda^{\sum_{i=1}^{n} t_n}}{t_n!}\right]$$

$$\frac{\partial l}{\partial \lambda} = -n + \frac{1}{\lambda}\sum_{i=1}^{n} t_i \tag{14}$$

4. Case Study

4.1 Type and Source of Data

The data used in this research were secondary data, the data of earthquake in Nabire, Papua in November 28–December 27 of 2004. The data were obtained from the Meteorological, Climatological and Geophysical Agency of Class II Geophysical Station of Angkasapura. Then, the data testing was conducted using Chi-Square.

If the steps conducted in determining the parameters of Poisson Distribution above are implemented to all data, the results will be as follows:

Table 1 Poisson Distribution Value

Hour	Poisson Parameter	Energy (Joule)	Richter Scale	Mean	Variance
	Lambda				
09.00 - 15.00	51.931	66.2514	5.941	51.931	51.931
15.00 – 21.00	49.0345	68.9397	5.9676	49.0345	49.0345
21.00– 03.00	46.5517	71.4674	5.9916	46.5517	46.5517
03.00 – 09.00	42.9655	75.5502	6.0286	42.9655	42.9655

If the value of (*t* = frequency data of aftershock) of each hour is substituted into Survival Function, Probability Density Function (PDF), Cumulative Distribution Function (CDF) and Hazard Function, the form of the graphs will be as follows:

Graph of Poisson Distribution for the Period (03.00 – 09.00 and 15.00 – 21.00)

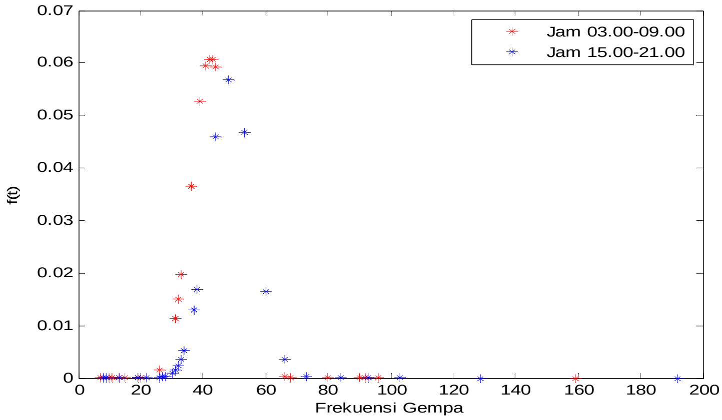

Figure 1 Poisson PDF Graph

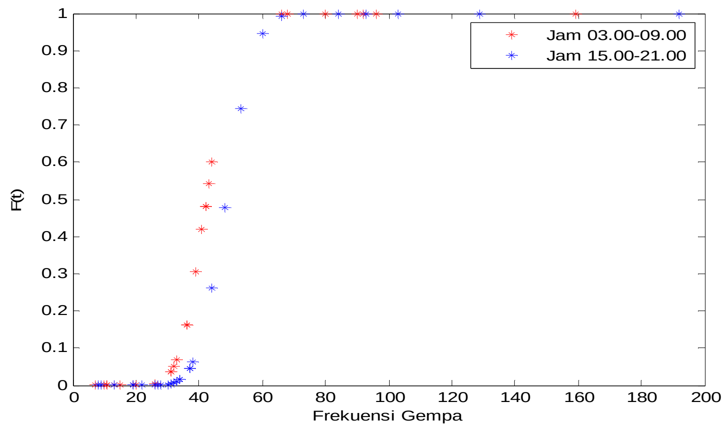

Figure 2 Poisson CDF Graph

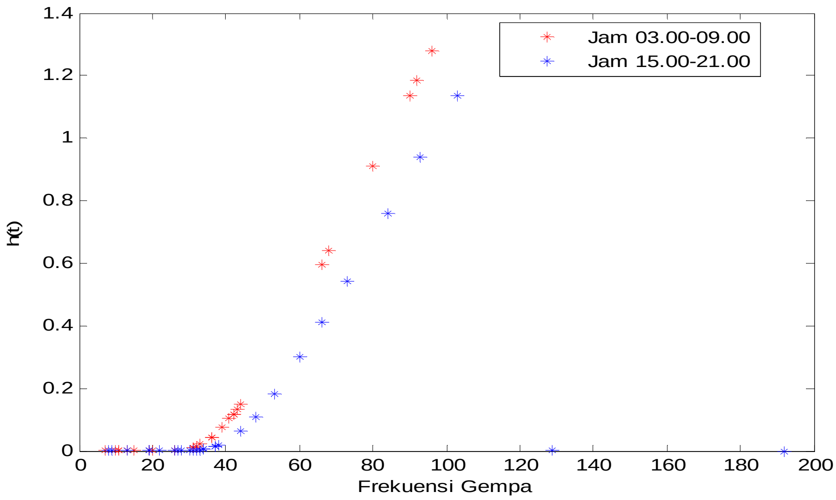

Figure 3 Poisson Hazard Graph

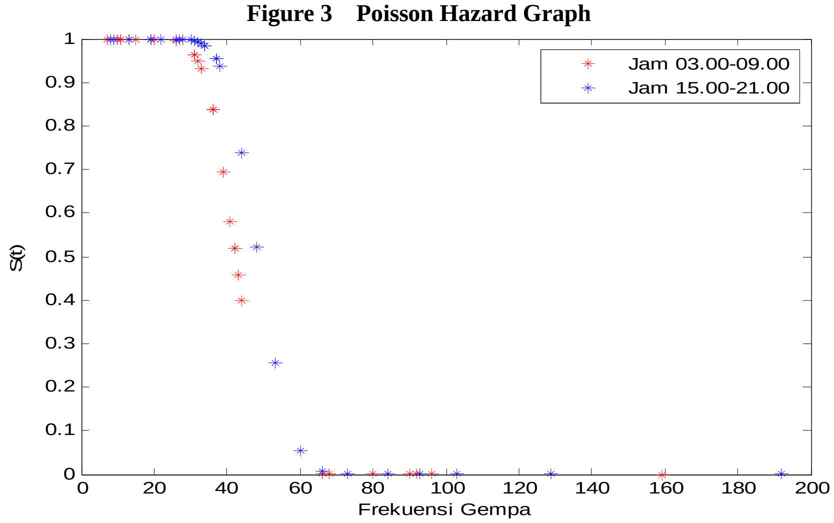

Figure 4 Poisson Survival Graph

According to the Graphs of Survival Function, Probability Density Function (PDF), Cumulative Distribution Function (CDF), Hazard Function of Poisson Distribution and to Table 1, it is clearly seen that there is a difference in the frequency of aftershock for each period of hour. For example, in the graph of survival function (Figure 4) of aftershock in November 28/29 of 2004 in the period of 03.00–09.00, with a magnitude of 6.0286, it can be interpreted as a probability of the frequency of aftershock higher than 159 for 0.0045. Then, in Figure 2, it can be interpreted as a probability of frequency of aftershock lower than 159 for 0.9955. While in Figure 3, it shows the reduction in low-frequency earthquake in the early of November 28/29, but it increased in the following days. Other things that can be analyzed are the magnitudes of energy and earthquake resulted. Thus, it also can be seen that the period of 03.00–09.00 has higher frequency of earthquake compared with other periods in which the parameter λ = 42.97, mean = 42.97, magnitude (M) = 6.03, richter scale, energi (E) = 75.55 Joule and type VII mercalli intensity were recorded. Thus, if the aftershock hit Nabire, Papua, in 2004 in the period of 03.00–09.00 occurred around the settlements, it was estimated that it would make all people coming out from room, even would be felt by people driving a car, and would make the bad-constructed buildings and chimneys collapsed.

5. Conclusion and Suggestion

5.1 Conclusion

Based on table 4.1 and the graphs resulted by, it is clearly seen that the period of 03.00–09.00 has the higher frequency of earthquake compared with the periods of 15.00–21.00, 21.00–03.00 and jam 09.00–15.00. It is seen from the richter scale resulted for 6.0253 and the magnitude of energy for 75.55 Joule.

5.2 Suggestion

In this research, the estimation of Poisson paremeter, lambda$^{\lambda}$, used the Maximum Likelihood Estimation (MLE) method. It is expected that further research can be conducted by using different methods.

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